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AN
ANALYSIS
OF A
COURSE OF LECTURES
ON THE PRINCIPLES OF
NATURAL PHILOSOPHY,
READ IN THE UNIVERSITY OF CAMBRIDGE.

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BRITISH MUSEUM

TO

THE SOCIETY OF TRINITY COLLEGE

IN THE UNIVERSITY OF CAMBRIDGE,

THE FOLLOWING PLAN

OF A COURSE OF LECTURES

ON THE PRINCIPLES OF

NATURAL PHILOSOPHY,

READ DURING SEVERAL SUCCESSIVE YEARS

IN THE OBSERVATORY OF THE COLLEGE,

IS RESPECTFULLY INSCRIBED

AND DEDICATED

BY

April 18, 1784.

THE AUTHOR.

THE following analyſis was originally drawn up for the uſe of the ſtudents and other members of the Univerſity of Cambridge, who attended the Author's courſes of Lectures on the Principles of Natural Philoſophy.

All the copies of the former impreſſion being diſpoſed of, it has been reviſed for publication, and is now reprinted with conſiderable additions.

Explanatory notes are ſubjoined to moſt of the articles with ſuch demonſtrations as could be made intelligible without figures, and admitted of being compriſed in a ſmall compaſs.

The additional propoſitions inſerted in this edition are chiefly in Mechanics, Hydroſtatics, *and Aſtronomy. In Optics, the deſcription of experiments on the phenomena of the Sun's image formed by the ſolar rays flowing through apertures of different figures and dimenſions, has been added, with ſome few other articles.*

A ſmall ſet of Tables is given in the Appendix, calculated to facilitate the computations which occur in applying ſome of the aſtronomical propoſitions to practice.

AN
ANALYSIS, &c.

PRINCIPLES OF STATICS AND MECHANICS, WITH INTRODUCTORY ARTICLES.

ARTICLE I.

IF light bodies float upon the ſurface of water contained in any veſſel, they are obſerved to be attracted toward the ſide of the veſſel.

Thin glaſs bubbles, an inch or two inches in diameter, are well adapted for making this experiment. Small pieces of cork alſo may be uſed. It is obſervable, that the floating bodies will frequently tend toward each other, as well as toward the ſide of the veſſel. Their motion is accelerated, as we might expect, from the action of a force which operates conſtantly on them whether it is uniform or otherwiſe. The ſame phenomena may be obſerved in the bubbles floating on the ſurface of water or other liquor, when their tenacity is ſuch as will give them ſufficient conſiſtence.

That power (whatever be the cauſe of it) by which bodies are impelled towards each other, has been termed by Sir I. Newton and others attraction, gravity, or gravitation. It is an univerſal principle inherent in all bodies or as far as human obſervation extends.

II.

Glaſs planes held contiguous attract the water in which they are immerſed, and raiſe it up between them above the level of the external water.

If the planes are held parallel, the water will riſe every where nearly to the ſame heighth; but if they are inclined at a ſmall angle, the ſection of the planes being vertical, the heights of the water at different diſtances from the ſection will be leſs, in the ſame proportion as the diſtances are greater. A line paſſing along the ſurface of the elevated water is an Hyperbola.

III.

Capillary tubes attract the water in which they are immerſed, and raiſe it above the level of the external water.

The elevation of water in capillary tubes, and between the glaſs planes in the laſt article, is occaſioned by the power of attraction which glaſs exerts on water. In any glaſs veſſel or tube which is not capillary, the water contained in it is always obſerved to be a little elevated at the ſides of the veſſel: in capillary glaſs tubes alſo the rim or circumference adjacent to the water's ſurface being the cauſe of its elevation, the quantity elevated will be proportioned to this circumference or to its diameter; and being at the ſame time as the ſquare of the diameter multiplied into the height, it follows that the height to which the water is elevated will be inverſely as the diameter of the capillary. It appears therefore that in any glaſs capillary tube the height to which it will elevate water, and keep it ſuſtained, multiplied into the diameter of the capillary, is a given quantity; this is found by trial to be .053 part of an inch: by means of this value, the diameter of a capillary tube being given, the height to which it will elevate water will be known; for it will be the quotient of .053 divided by the diameter: thus, ſuppoſe the diameter is $\frac{1}{20}$ of an inch, the height to which the water will be elevated $= .053 \times 20 = 1.06$.

The moſt exact way of meaſuring the diameter of a capillary tube is, by incloſing ſome mercury in it; if a given quantity of mercury always occupies the ſame length of the tube as it is moved along to different parts, the tube is known to be cylindrical. Having meaſured the length of a column, or of ſeveral columns for greater exactneſs, let the length of the ſum of theſe columns be l in inches; let the mercury which has been included, be accurately weighed, and let w be the weight of it in grains; if 13.6 be the ſpecific gravity of mercury (which is that of the pureſt ſort) when that of water is 1, the diameter of the capillary will be $\sqrt{\frac{w}{l}} \times .01923$.

IV.

Plain ſurfaces of hard bodies brought into contact, ſtrongly cohere.

If a number of equal ſpheres touch each other, the quantity of ſurface in contact will be decreaſed, by diminiſhing the ſphere's diameters; the points of contact between equal ſpheres being in the direct ratio of their diameters. Hence if the force of coheſion depends on the quantity of ſurface in contact, a ſubſtance conſiſting of equal ſpheres, which are perfectly hard and ſmooth, and of evaneſcent diameters, will oppoſe no reſiſtance to the motion of bodies which impinge on, or move through them, except that which ariſes from the inertia of the parts diſplaced: and this will afford an idea of a perfect fluid.

V.

A copper wire, one-tenth of an inch in diameter, will ſuſtain a force equal to near 300 lb. before it is broken.

Metallic bodies loſe their coheſion of parts entirely, by being expoſed to certain degrees of heat; but on cooling, recover that quality without ſenſible diminution. May not heat operate by changing the figures of the component parts, ſwelling them into roundneſs, from being terminated by flat ſurfaces? It is certain that in the latter caſe, the quantity of ſurfaces in contact would be in a phyſical ſenſe infinitely greater than in the former: ſo that if the coheſion of parts depends on the quantity of contact, heat applied in due intenſity, will be ſufficient to account for the converſion of a ſolid body into one that is fluid, only by changing the figures of the component parts.

PRINCIPLES OF STATICS.

VI.

If a body be impelled by two or more forces and continues at reſt, thoſe forces act equally in oppoſite directions.

The theory of forces conſiſts of Statics, or the principles of equilibrium, and Mechanics, or the principles of motion. In Statics, the impelling forces only are conſidered, the inertia of the body impelled, being of no conſequence; when an equilibrium is produced by the action of oppoſite forces, the quantity of matter urged by them need not be taken into account; but in Mechanics both the motive forces, and the inertia of the bodies moved muſt be eſtimated in order to aſcertain the motion produced.

Statics precede Mechanics in order, becauſe when it is required to aſſign the motion produced by any number of forces acting at once on a body, it muſt be firſt aſcertained what portion of theſe forces ſuſtain each other in equilibrio; the direction and quantity of the remaining force will give the direction in which the body will be moved, and the force by which it is impelled in that direction.

VII.

If a body be acted on by two forces repreſented in quantity and direction by the two ſides of a parallelogram, a force oppoſed in the direction of the diagonal, and proportional to it in quantity will counterpoiſe the other two, and the body will remain at reſt.

From this ſtatical principle we obſerve that any two forces may be compounded into one, and converſely any force may be reſolved into two; but though two forces, of which the quantities and directions are given can be compounded into one force, and one force only, yet, any given force may be reſolved into two others, which may be varied infinitely as to their quantity and direction.

VIII.

If a body be acted on by three forces repreſented in quantity and direction by the three ſides of a triangle, or by four forces repreſented in quantity and direction by the four ſides of a trapezium, the body thus acted upon will remain at reſt.

Theſe propoſitions admit of very exact illuſtration from matter of fact; when the weights uſed amount to ſome thouſands of grains, the addition of two or three grains to any of the weights above their due quantity ſenſibly diſturbs the equilibrium.

IX.

If a rod be ſuſpended ſo as to move freely on an horizontal axis paſſing through the middle point of the rod, and two equal weights be applied at equal diſtances from the axis, they will exactly counterpoiſe each other.

The rod or lever here uſed is ſuppoſed in theory to be void of weight and inflexible. In making mechanical experiments, ſome difficulty occurs from the weights of the inſtruments uſed; ſince it is impoſſible to find any ſubſtance void of gravity. To obviate this, the weights of the materials muſt be previouſly counteracted by ſome equal and oppoſite force, ſo as to exactly balance them; thus in the experiment juſt deſcribed, the weight of the rod or lever is counteracted by making it reſt on a fulcrum or axis, by which means, whatever weights are afterwards applied to produce an equilibrium, they will operate freely being uninfluenced by the weight of the lever.

X.

A weight of two ounces ſuſpended on the arm of a lever, will balance a weight of four ounces ſuſpended on the other arm at one half the former diſtance from the axis.

There will be always an equilibrium on the ſtrait lever when the weights ſuſpended from the oppoſite arms are inverſely proportional to their diſtances from the axis.

XI.

There will be an equilibrium on a ſtrait lever when the ſums of the products formed by multiplying each weight into its reſpective diſtance from the axis, are equal on both ſides of the axis.

From this principle we obtain a method for finding ſuch a point on the arm of the lever impelled by weights acting at different diſtances from the axis, that the weights being all applied together at this point ſhall have preciſely the ſame effect as when they act at their reſpective diſtances. Let the weights be p, q acting at the diſtances from the axis a, b reſpectively: then the ſum of the products formed by multiplying each body into its diſtance from the axis will be $ap+bq$: now let both the bodies be applied together at the diſtance $\frac{ap+bq}{p+q}$; then this diſtance being multiplied by the ſum of the weights $p+q$ will become $ap+bq$ the ſame as before, by which it is known that the ſame force is exerted on the arm of the lever, whether the weights p, q are applied at the diſtance a, b reſpectively, or both weights $p+q$ are together applied at the diſtance $\frac{ap+bq}{p+q}$.

In the ſame manner let the weights which act on the arm of the lever be p, q, r, s, applied at the diſtances a, b, c, d reſpectively; then their force to move the lever will be exactly the ſame as when the ſum of the weights $p+q+r+s$ is together applied at the common diſtance $\frac{ap+bq+cr+ds}{p+q+r+s}$.

The point thus determined is called the centre of gravity of the weights or forces, p, q, r, and s; and from ſimilar principles when the weights are diſpoſed on different lines or planes, a point may always be aſcertained, in which if the gravity of the whole be concentrated, it will have the ſame effect in impelling the body to which the forces are applied, as when they all act in their reſpective places, the following conditions being obſerved: that the forces act in directions parallel to each other, and parallel to the direction in which they are all applied at the common centre of gravity.

The earth's gravity attracts each individual particle of a body: and the ſum of theſe weights will have the ſame effect on the body as if they were all together applied at its centre of gravity. This property is extended to bodies of every ſhape and dimenſion, and ſituated in every poſition in reſpect of the horizon, provided the earth's gravity may be conſidered to act on each particle in parallel lines, and to ſuffer no ſenſible alteration at different diſtances from the earth's centre. The neceſſity of this limitation is fully explained in a treatiſe on electricity by the right hon. lord Mahon.

The eſtimation of the forces by which gravity impells bodies near the earth's ſurface is much facilitated by this principle: for the gravity of any body, or ſyſtem of bodies, is by means of it transferred to a ſingle point, in which the whole is concentrated. It likewiſe mathematically expreſſes the diſtance of one body from another as far as regards the action of their reſpective weights, this diſtance being the ſame as that between their centres of gravity.

XII.

If a body hangs freely from a centre of ſuſpenſion, it will not be at reſt unleſs the line of direction produced paſſes through the centre of ſuſpenſion.

Gravity acts in a direction perpendicular to the horizon: and the whole weight of a body being collected into the center of gravity, that point will always endeavour to deſcend in a vertical line, and with a force equal to the bodies weight; for this reaſon a vertical line paſſing through the bodies centre of gravity is called the line of direction. When the line of direction does not paſs through the centre of ſuſpenſion, the bodies weight may be always reſolved into two forces, one of which impels the centre of gravity directly from the centre of ſuſpenſion, and is wholly counteracted, becauſe the centre of ſuſpenſion is fixed: the other force acting in a direction perpendicular to the former, and having no counterpoiſe, urges the centre of gravity, in the direction of a tangent to the circle which that point deſcribes round the centre of ſuſpenſion. It appears that this force is always as the ſine of the angular diſtance of the centre of gravity from the loweſt point; when the centre of gravity is in this loweſt poſition, the line of direction paſſes through the centre of ſuſpenſion, in which caſe the force that communicates motion to the centre of gravity vaniſhes.

If a body is ſuſpended by a line from a fixed centre, it will be at reſt only when the line coincides with a perpendicular to the horizon. An inſtrument of this kind is called a plumb-line, and is of very extenſive uſe both in philoſophy and the mechanic arts; by means of this line, the vertical direction and the horizontal plane which is perpendicular to it are preciſely determined.—The ſame may be effected by the level; but the plumb-line has the advantage of requiring no adjuſtments, whereas the exactneſs of the level depends on the poſition, either of the points from which the level is ſuſpended, or of the baſe on which it reſts in reſpect of the upper ſurface of the tube; and is beſides liable to ſome imperfections from which the plumb-line is free.

XIII.

A body, or ſyſtem of bodies, will not be at reſt, unleſs their centre of gravity is ſupported.

In the laſt article the centre of gravity was ſupported by a fixed centre of ſuſpenſion, immediately under which the centre of gravity reſted quieſcent. But there are many caſes in which the centre of gravity may be ſupported quieſcent, although it is not abſolutely attached to a centre of ſuſpenſion. In a ſyſtem compounded

of

of any differently ſhaped bodies, let a line be drawn from the common centre of gravity perpendicular to any ſurface contained in the ſyſtem; then if the point on which the ſurface and line interſect each other, although not fixed on a fulcrum, is ſo placed on it, that the centre of gravity may be underneath, the whole will remain quieſcent; even if the ſurface be abſolutely a plane, and perfectly ſmooth, and the fulcrum a point.

If the line drawn from the centre of gravity be not exactly perpendicular to the ſurface through which it paſſes, yet if there is a certain coheſion between the fulcrum and the ſurface which reſts on it, there will ſtill be an equilibrium. Many experiments on the centre of gravity are derived from this principle.

XIV.

If any ſyſtem of bodies be placed on a fixed point or fulcrum, it will not reſt in any poſition whatever, unleſs the centres of ſuſpenſion and gravity coincide.

If the centres of ſuſpenſion and gravity do not coincide, there will be innumerable poſitions of the latter point, in which the line of direction does not paſs through the centre of ſuſpenſion; in any of theſe caſes, the body or ſyſtem of bodies cannot be quieſcent, by Art. (XII).

XV.

If a body be ſuſpended from centres of ſuſpenſion paſſing through different points of the body, the interſection of lines drawn from theſe centres perpendicular to the horizon when the body is quieſcent will coincide with the centre of gravity.

In bodies that are of uniform denſity and at the ſame time admit of geometrical menſuration, the poſition of the centre of gravity may be aſcertained from theory: but in bodies wholly irregular, recourſe muſt be had to experiment, in order to determine this point.

XVI.

If the plane baſe of any body, is placed on an horizontal ſurface, the line of direction paſſing without the baſe, the body will fall: bnt if the line of direction paſſes through the baſe, the body will remain ſupported.

This is one among the many ways in which the centre of gravity may be ſuſtained. While the line of direction paſſes through the baſe, the bodies weight only preſſes the baſe more forcibly againſt the ſurface on which it reſts: when the line of direction paſſes without the baſe, the centre of gravity loſes its ſupport, and the body falls till the centre of gravity is ſuſtained.

XVII.

Let a ſolid of any kind be placed on an inclined plane: if the line of direction always paſſes without the baſe, and the friction of the ſurfaces be ſufficiently great, the bodies will roll down the plane; if the friction is diminiſhed to a certain degree, the bodies will deſcend partly by rolling, and partly by ſliding: if it were poſſible to re-move friction entirely, they would deſcend by ſliding without any rotation.

It ſeems at firſt view remarkable that a ſphere ſhould deſcend along an inclined plane without rolling; but it is certain that there is no force to produce the rotation except that which ariſes from the friction of the ſurfaces, which being entirely removed the ſphere muſt ſlide.

XVIII.

If a body with a flat baſe is placed on an inclined plane, and the line of direction paſſes through the baſe, the body will be quieſcent or ſlide down the plane, according as the coheſion of the ſurfaces is to the bodies weight in a greater or leſs proportion, than that which the ſine of the planes elevation bears to radius.

XIX.

If a rod or beam be ſo conſtructed that the line drawn through the centres of gravity and ſuſpenſion ſhall be perpendicular to any given line in the rod (for example, that which paſſes along its upper ſurface), the rod hanging freely on its axis of ſuſpenſion, will not be at reſt except when that given line is in an horizontal poſition.

This is plain becauſe in the poſition deſcribed, and in that poſition only, the line of direction paſſes through the centre of ſuſpenſion.

It is evident that when the rod is quieſcent, the centre of gravity will be immediately under the axis of ſuſpenſion.

Upon this principle the balance, an inſtrument of very general and extenſive uſe, is conſtructed. It is applied to eſtimate the weights of bodies, that is, to compare the weight of any given ſubſtance with certain ſtandard weights previouſly determined.

When the axis of motion paſſes through the centre of gravity, the rod will reſt in any poſition whatſoever: If two weights were ſuſpended from the extremities of a beam or rod thus conſtructed and at equal diſtances from the axis, it is certain that the quieſcence of the rod would denote an equality between the weights acting upon it: and in this caſe the rod would be quieſcent in any poſition; but it has been found more convenient that ſome fixed poſition of the rod ſhould denote the equality of the weights, and none is ſo eligible as that which is commonly uſed, the balance being horizontal when the weights ſuſpended from the extremities of the arms are equal: for practical uſe this poſition is aſcertained by an index directed to ſome fixed point.

The index is uſually in a continuation of the line which paſſes through the centre of gravity and the axis of ſuſpenſion perpendicular to that axis.

XX.

When the balance is quieſcent, if the index is directed to any fixed point; and, when the balance is reverſed, the index is directed to the ſame fixed point, the index muſt be in the ſame line with the centre of gravity, and the axis of ſuſpenſion.

By this means the index is adjuſted in the line which joins the centre of gravity and the axis of ſuſpenſion.

The line which joins the points from which the weights are ſuſpended at the extremities of the arms ſhould paſs through the centre of gravity, perpendicular to the line which joins the centres of gravity and ſuſpenſion; although in ſome conſtructions it approaches, or even coincides with the axis of ſuſpenſion: however this may be, the lengths of the arms will be eſtimated by the perpendicular diſtances between the points at the extremities of the arms to which the weights are ſuſpended, and the line joining the centres of gravity and ſuſpenſion.

The weights of bodies are referred to known ſtandards of various kinds: but as philoſophical computations are chiefly made in decimal fractions, it is found extremely convenient to arrange weights in that progreſſion. There ſhould be two ſetts of them, one for the eſtimation of larger weights, and the other for that of the ſmaller.

The following may be found ſufficient for moſt philoſophical purpoſes.

wts.		oz.	wts.		gr.
4	each of	100	1	of	1000
1	of	50	1	of	500
4	of	10	4	of	100
1	of	5	1	of	50
4	of	1	4	of	10
1	of	.5	1	of	5
4	of	.01	4	of	1
1	of	.005	1	of	.5
4	of	.001	4	of	.1
			1	of	.05
			4	of	.01

In the firſt progreſſion of weights a Troy ounce is aſſumed an unit. In the latter ſett of weights, unity denotes a grain. By help of theſe progreſſions continued further if neceſſary, we may expreſs with the greateſt readineſs the weight of any ſubſtance decimally to 4 or 5 places of figures including the integers. Thus a ſubſtance which would be found by the uſual method of weighing to be 1 oz. 2 pennyweights and 20 grains, would appear by the firſt ſett of weights to be =1.142 oz. which requires no further reduction.

XXI.

Let A and B, denote the arms of a balance, and let a weight *a* be ſuſpended from the arm A, balance another weight *b* ſuſpended from the arm B: if *b* ſuſpended from A balances *a* ſuſpended from B, the arms of the balance are equal; and the weights *a* and *b* are alſo equal.

By this means the arms of a balance are adjuſted to equality.

When the arms of a balance are equal, it is indifferent from which arm the body to be weighed is ſuſpended.

When the arms are unequal, the balance is ſaid to be falſe, as not giving the true weight of the body weighed, whether it be ſuſpended from the ſhorter arm or from the longer. There are however ſeveral properties of the falſe balance which are extremely uſeful in the eſtimation of weights, as well as in correcting the errors which may have ariſen in the adjuſtment of the true balance.

XXII.

A weight which counterpoiſes an ounce when ſuſpended from the longer arm of a falſe balance being added to the weight which counterpoiſes an ounce ſuſpended from the ſhorter arm, the ſum of theſe two weights is always greater than two ounces.

The exceſs is that part of an ounce, which is expreſſed by a fraction of which the numerator is the ſquare of the difference of the arms, and the denominator, the product of the arms. Let x and y be the two erroneous weights, x being leſs, and y greater than an ounce, and let the arms of the falſe balance be in the proportion of $a : b$, a being leſs than b: then $x = \frac{a}{b}$, and $y = \frac{b}{a}$ wherefore $x + y = \frac{a^2 + b^2}{ab}$; from which, if 2, the true ſum of the weights be ſubtracted, there will remain, the exceſs of the falſe, above the true weights, $= \frac{a^2 + b^2}{ab} - 2 = \frac{a^2 + b^2 - 2ab}{ab} = \frac{\overline{b - a}^2}{ab}$.

This property is eaſily illuſtrated by a balance with graduated arms. Suppoſe the lengths of the arms to be 12 and 8, an ounce ſuſpended from the arm 12 will counterbalance $1\frac{1}{2}$ oz ſuſpended from the arm 8; and an oz. placed at 8 will balance $\frac{2}{3}$ oz, ſuſpended from 12: the ſum of theſe falſe weights $= 1\frac{1}{2} + \frac{2}{3} = 2\frac{1}{6}$ the exceſs of the falſe above the true weights being $\frac{1}{6}$: to compare this with the general ſolution we have $a = 8$, $b = 12$, and $\overline{b - a}^2 = 16$, $ab = 96$, and the exceſs $= \frac{\overline{b - a}^2}{ab} = \frac{16}{96} = \frac{1}{6}$ as was before found.

XXIII.

If any ſubſtance be weighed ſucceſſively being ſuſpended from the longer and ſhorter arms of a falſe balance, the true weight will be a geometrical mean between the two falſe weights.

Suppoſe the true weight of the ſubſtance weighed to be w, and the two falſe weights, x and y, of which x is leſs than y; and let a and b be the arms of the balance, a being leſs than b, then we have $x : w :: a : b$
and $y : w :: b : a$
joining theſe ratios $xy : w^2 :: ab : ab$
it appears that $x : w : w : y$.

$$\begin{array}{r} x : w :: a : b \\ y : w :: b : a \\ \hline xy : w^2 :: ab : ab \end{array}$$

There

Thus if the arms are 12 and 8, and the true weight of the ſubſtance weighed 1 oz, the two falſe weights will be $\frac{3}{2}$ oz. and $\frac{2}{3}$ oz. between which a mean geometrical proportional is 1 ounce, the true weight.

It appears that equality of the arms is not neceſſary for the eſtimation of weights, the rule juſt demonſtrated affording means for finding the weights of bodies with the ſame exactneſs, when the arms are unequal, as when they are perfectly adjuſted to equality. The trouble of taking a geometrical mean is avoided when the arms are nearly equal, becauſe in this caſe the arithmetical mean is to all practical purpoſes equal to it. Thus ſuppoſe the two falſe weights were 564 grains and 566 grains; the arithmetical mean between them is 565, the geometrical mean being 564.999 that differs from the other by too ſmall a quantity, to be at all ſenſible in practice.

By means of the balance it is found that equal magnitudes of the ſame kind of uniform ſubſtance endeavour to deſcend with equal forces, or which is the ſame thing are of equal weights. Conſequently the weights of ſubſtances that are uniform and of different magnitudes will be as their magnitudes, or as the quantities of matter contained in the ſubſtances. For example, a cube of braſs, two inches each way, will weigh eight times as much as a cubic inch of the ſame ſubſtance.

Suppoſe the cube of two inches each way, were compreſſed into the compaſs of a cubic inch, the weight and quantity of matter will remain unaltered; by a due application of this reaſoning it follows in general, that in ſubſtances of every kind, their weights are proportional to the quantities of matter contained in them.

By the balance it is found that different kinds of ſubſtances which have the ſame magnitudes, are of different weights; which weights will therefore give the proportion of the quantities of matter contained in the ſubſtances. Hence is derived the definition of denſity, which is a relative term ſignifying the proportion which the weight or quantity of matter in one body bears to that in another when their magnitudes are equal: thus, a cubic inch of braſs weighs eight times as much as a cubic inch of water; or in other terms the denſity of braſs is eight times greater than that of water. In general the quantities of matter contained in bodies are as their weights or as their denſities and magnitudes jointly; the denſities are as their weights directly, and magnitudes inverſely; And their magnitudes are as their weights directly, and denſities inverſely.

XXIV.

There will be an equilibrium on a ſtrait lever of any kind when the power is to the weight, as the diſtance of the weight to the diſtance of the power from the fulcrum. A lever is ſaid to be of the firſt kind when the fulcrum or axis of ſuſpenſion is between the power and the weight. It is of the ſecond kind when the weight is between the power and fulcrum. A lever is ſaid to be of the third kind when the power is between the fulcrum and weight.

In making experiments on the Mechanic powers, the weights of the materials they are compoſed of, are ſuppoſed perfectly balanced among themſelves before the weights and powers are applied; otherwiſe they will have ſome effect in altering the proportion of the power to the weight, when there is an equilibrium, and muſt be allowed for accordingly.

XXV.

There will be an equilibrium on a bent lever of any ſhape and dimenſion, when the power and weight are inverſely proportional to lines drawn from the fulcrum, perpendicular to the lines of direction in which the power and weight are applied.

Here the lines of direction are ſuppoſed to be drawn through the points on the arms of the lever where the power and weight are applied, whether they are perpendicular to the horizon or otherwiſe, provided they are always in the ſame plane with that in which the lever moves.

XXVI.

On a fixed pully there is an equilibrium when the power is equal to the weight.

Although no Mechanical advantage is obtained by the fixed pully, yet it is very uſeful for changing the direction of any force without altering its quantity. Thus in a given weight may by a fixt pully be made to act horizontally or in any direction inclined to the horizon.

XXVII.

On a ſingle moveable pully there will be an equilibrium, when the power is to the weight, as one to two.

The weight of the moveable pully muſt be either firſt balanced before the power and weight are applied, or it muſt be included in the weight ſupported.

XXVIII.

There will be an equilibrium in a ſyſtem of pullies, in which the ſame ſtring goes round all the pullies contained in two blocks, when the power is to the weight, as unity is to the number of ſtrings at the lower block.

XXIX.

On a ſyſtem of moveable pullies, in which a ſeparate ſtring goes round each pully, there will be an equilibrium when the power is to the weight, as unity to that power of two of which the index is the number of moveable pullies.

It is to be obſerved that in all theſe ſyſtems, the pullies uſed to ſuſtain a given weight, act againſt the force or powerapplied. In the ſyſtem which is next deſcribed all the moveable pullies act by their weight ſo as to increaſe the power applied.

XXX.

In that ſyſtem of pullies in which the ſtring that goes round each pully is fixed to the weight, there will be an equilibrium when the power is to the weight, as unity is to that power of two, of which the index is the number of pullies, the whole being decreaſed by unity.

A given power when applied to this ſyſtem of pullies will ſuſtain a greater weight, than when the ſame power is applied to an equal number of pullies diſpoſed acording to any other ſyſtem.

XXXI.

When the reſiſtance or weight which acts againſt a wedge is perpendicular to the ſides, there will be an equilibrium, if the power be to the weight, as the baſe of the wedge to the ſum of the ſides; or as the ſine of the ſemi-angle at the vertex of the wedge to radius.

This is the caſe uſually adopted for making experiments on the wedge.

When the direction of the reſiſtance is inclined to the ſides of the wedge at an angle greater or leſs than 90°, the proportion between the power and the weight, when there is an equilibrium, will depend on the manner in which the reſiſtance is applied to the two ſides.

In the firſt place, let the reſiſtance or weight be ſuch as may act on the ſides by means of ſome rigid pointed body which will adhere to the place at which it is applied without ſliding: let the ſemi-angle at the vertex of the wedge be A; the angle at which the reſiſtance is inclined to the ſides of the wedge $= B$: there will be an equilibrium when the power is to the weight as the coſine of $B - A$ to radius. When therefore the direction of the reſiſtance or weight is perpendicular to the baſe, becauſe $B = A$, the power will be equal to the weight. If it acts in a direction parallel to the baſe, the power will be leſs than the weight in a greater than any aſſignable proportion. If the reſiſtance is applied in a direction perpendicular to the ſides, the power will be to the weight as the baſe of the wedge to the ſum of the ſides, when they ſuſtain each other in equilibrio.

Secondly; If the reſiſtance is applied by means of any ſubſtance that may ſlide along the ſides, the proportion of the power to the weight will be altered; being now as ſine A × ſineB to R^2 (R being put for the radius) in this caſe, if the reſiſtance acts in a direction perpendicular to the baſe of the wedge, the power will be to the weight or reſiſtance as $\overline{\text{ſine A}}^2$ to R^2 : if the reſiſtance is applied in a direction parallel to the baſe, the power will be to the weight as ſine A × coſ. A to R^2 that is as the ſine of the angle at the vertex of the wedge to diameter. Laſtly; when the reſiſtance acts in a direction perpendicular to the ſides, the power will be to the weight or reſiſtance as the baſe of the wedge to the ſum of the ſides when they are in equilibrio.

Due regard being had to theſe diſtinctions, it is found that experiments on this mechanic power perfectly correſpond with the theory, whatever be the direction of the forces in reſpect of the ſides to which they are applied in order to ſuſtain the weight.

XXXII.

There will be an equilibrium on the wheel and axle when the power is to the weight, as the radius of the axle to the radius of the wheel.

XXXIII.

There will be an equilibrium on the ſcrew when the power is to the weight, as the diſtance between two contiguous threads to the circumference which the power deſcribes.

XXXIV.

If a weight is ſuſtained on an inclined plane, by a power which acts in a direction parallel to the plane, the power will be to the weight, as the height of the plane to the length.

XXXV.

If a weight is ſuſtained on an inclined plane by a power which acts in a direction parallel to the baſe, the power will be to the weight as the height of the plane to the baſe.

XXXVI.

If a weight be ſuſtained on an inclined plane by a power which acts in a direction any how inclined to the plane, the power will be to the weight as the ſine of the plane's elevation, to the coſine of the angle in which the direction of the power is inclined to the plane.

XXXVII.

On a compound lever there will be an equilibrium when the power is to the weight as the product of the powers on each lever is to the product of the weights.

There will be an equilibrium on a machine compounded of any number of Mechanic powers, when the ratio of the power to the weight, is equal to the ſum of the ratios expreſſing the power and weight, when there is an equilibrium on each mechanic power.

The exactneſs of ſtatical experiments muſt by no means be judged of only from the equilibrium which is produced by the action of oppoſed forces.

Coheſion and a defect in the mechanical conſtruction operate to prevent motion, and thus create the appearance of an equilibrium, between weights or other forces, whether they are in their true proportion or otherwiſe. That experiments on the equilibrium of bodies may be ſatisfactory, the effects of coheſion and of mechanical imperfections ought to be removed as much as poſſible; and the remains of them aſcertained by obſerving what ſmall weight is neceſſary to overcome them. Limits alſo ſhould be aſſigned to any errors that can happen in adjuſting the directions in which the different forces are applied.

In experiments on the motion of bodies the effects of friction and inaccurate conſtruction, appear immediately by altering the ſpaces deſcribed, times of deſcription and velocities acquired by bodies which are the objects of experiments. The inertia of the materials which the inſtruments uſed conſiſt of muſt be allowed for, and the air's reſiſtance taken into account in thoſe caſes where the velocity acquired is conſiderable. When all impediments to motion are removed or reduced within certain limits, the experiments will be the more deciſive and the more nearly coincident with the theory of motion, according as theſe limits approach more nearly to coincidence.

PRINCIPLES OF RECTILINEAR MOTION.

XXXVIII.

Every body perſeveres in its ſtate of reſt or uniform motion in a right line, untill a change is effected by the agency of ſome external force.

This is aſſumed as an axiom in Mechanics: it may be may be made evident to the ſenſes by very ſatisfactory experiments.

That power by which bodies endeavour to continue in the ſtate of quieſcence or uniform motion in a right line is called Inertia or ſometimes Vis Inertiæ.

XXXIX.

Bodies by their Vis Inertiæ, reſiſt the impulſes of any force tending to ſet them in motion. Bodies in motion reſiſt the impulſes of any force tending to augment or diminiſh their motion.

To eſtimate the motion of bodies, their inertia muſt be taken into account; whereas the equilibrium produced by the action of any forces depends only on the quantities and direction of theſe forces, being wholly independent of inertia.

To form an adequate idea of the laws that are obſerved in the communication of motion, the bodies impelled ſhould be conceived to exiſt in free ſpace and void of gravity, ſo that to a given ſubſtance various degrees of forces may be applied; but this cannot be effected in bodies falling freely near the earth's ſurface, becauſe the ſame ſubſtance is always impelled by the ſame force of gravity which admits not of increaſe or diminution.

This difficulty may be obviated by balancing two equal weights joined by a fine flexible line which goes over a pully: the axle of the pully muſt reſt on wheels conſtructed for the purpoſe of diminiſhing friction.

The motive force of gravity being deſtroyed by the contrary and equal action of the weights, they will remain quieſcent till ſome force is applied to them. When any impulſe is communicated in a vertical direction, they will afterwards be obſerved to deſcribe equal ſpaces in equal times, or in other words they will move uniformly, and the velocity communicated will be preciſely the ſame as if the ſame impulſe had been impreſſed on a quantity of matter equal to the two bodies exiſting in free ſpace without gravity; due allowance being made for the inertia of the wheels, according to rules hereafter deſcribed.

XL.

Let two equal weights be ſuſpended by a line joining them and going over a fixed pulley. If any weight be added to either of them, it will preponderate, and in its deſcent will deſcribe ſpaces which are as the ſquares of the times of falling from reſt.

If a body falls freely by the force of gravity, it will deſcribe about ſixteen feet and an inch in one ſecond and ſixty four feet in two ſeconds of time; theſe great velocities of bodies which deſcend freely make it impoſſible to obſerve the proportion of the ſpaces deſcribed in reſpect of the times with any preciſion. In the following experiment the quantity of matter continuing the ſame, the motive force of gravity is ſo diminiſhed that the velocity generated in a few ſeconds will be inconſiderable; and will admit of the ſpaces deſcribed, and times of deſcription being ſatisfactorily obſerved.

Suppoſe each of the two equal weights to be 26 m and the additional weight applied to either of them $= m$; then this additional weight will be the only force by which motion is communicated. The maſs moved will be $m + 52m$ + the Inertia equivalent to that of the wheels. In the inſtrument conſtructed for theſe experiments the inertia of the wheels was 11 m which makes the inertia altogether $= 64\,m$. The preponderating weight deſcending from quieſcence during the times 1, 2, and 3 ſeconds, deſcribes 3, 3×4, and 3×9 inches reſpectively, the ſpaces being in the duplicate ratio of the times of motion.

By means of the inſtrument juſt referred to, the force and the inertia or maſs moved may be conſtructed in any given proportion, under certain limitations not neceſſary to be here mentioned.

In the notes to theſe experiments the following abreviations will be uſed: the letter S, will ſignify the ſpace de-

deſcribed by any body of which the quantity of matter is Q, being moved by the force M, during the time T, the motion being eſtimated from quieſcence. s, t, w, m, will repreſent ſimilar quantities in reſpect of another body of which the maſs is q: the forces M and m, are ſuppoſed to act uniformly.

XLI.

If different forces are ſucceſſively applied to accelerate equal quantities of matter from quieſcence, the ſpaces deſcribed in any given time will be in the ſame proportion with the forces.

If a quantity of matter $= 64\,m$, is impelled by a force $= 64m$, that is by the whole force of gravity acting on it, the ſpace deſcribed from reſt in one ſecond of time will be 193 inches. If the ſame quantity of matter $64m$, is impelled for one ſecond by a force $= m$, the ſpace deſcribed during that time from quieſcence $= \frac{193}{64} =$ about 3 inches; being decreaſed in the ſame proportion with the force of impulſe. In general $\frac{S}{s} = \frac{T^2}{t^2} \times \frac{M}{m} \times \frac{q}{Q}$; and when $T = t$ and $Q = q$, as in the experiment, $\frac{S}{s} = \frac{M}{m}$.

XLII.

If the ſame force impels different quantities of matter for any given time, the ſpaces deſcribed from quieſcence will be inverſely as the quantities of matter moved.

If the force $= m$, and the maſs $= 64m$, the ſpace deſcribed during one ſecond will be 3 inches. If the force $= m$ and the maſs $= 32m$ the ſpace deſcribed in the ſame time will be 6 inches. In general when the forces and times of their action are the ſame $\frac{S}{s} = \frac{q}{Q}$.

XLIII.

If the force is increaſed or diminiſhed in the ſame proportion with the maſs moved, the ſpaces deſcribed from reſt, in the ſame time, will be equal.

Let m, $2m$, and $4m$, be the moving forces which impel the quantities of matter $32m$, $64m$, and $128m$, reſpectively; then m will impel $32m$ through 6 inches in a ſecond: and $2m$ will impel $64m$, and $4m$ will impel $128m$ through the ſame ſpace in the ſame time. The deſcents being eſtimated from quieſcence.

The general principle is this, $\frac{S}{s} = \frac{M}{m} \times \frac{q}{Q} \times \frac{T^2}{t^2}$; but $T = t$ and $M : m :: Q : q$, or $Mq = mQ$, by the conſtruction of the experiment; wherefore $S = s$.

From hence it is known that when different quantities of matter deſcribe equal ſpaces in equal times, being accelerated from quieſcence, the forces muſt be in the ſame proportion, with the quantities of matter moved.

This principle applies to the actual deſcent of bodies falling freely towards the earth's ſurface: for it is obſerved that though heavy and light bodies as they are uſually called falling from reſt in open air deſcend with very different degrees of velocity, yet in vacuo, their deſcent through any given ſpace will be performed in the ſame time; from whence the inference is, that the force exerted by gravity on falling bodies is in the ſame proportion with the quantities of matter contained in them.

XLIV.

If a piece of gold and a feather begin to fall at the ſame inſtant from the top of a receiver well exhauſted of air, they will both deſcend to the bottom of the receiver in the ſame time.

In a deſcent of 5 feet, no difference in the times of deſcent is perceptible.

In deſcribing greater ſpaces it is probable that the air remaining in the receiver, however ſmall in quantity, may cauſe ſome ſenſible retardation in the lighter body.

XLV.

If a body is moved from quieſcence during any given time, it will at the end of that time, have acquired ſuch a velocity as will, if continued uniform, carry it in the ſame time through double the ſpace which the body has already deſcribed to acquire that velocity.

Let the maſs moved be $64 m$, the force $= m$: then in two ſeconds the maſs will have deſcribed 12 inches with an uniformly accelerated motion: if at the end of that ſpace the force m be removed and ſo prevented from any further acceleration of the body, it will proceed uniformly and deſcribe 24 inches in the next two ſeconds of time.

By this means the velocities acquired by bodies that are acted on by conſtant forces admit of being obſerved experimentally with great preciſion.

XLVI.

If the ſame force acts on the ſame maſs, for different times, 1 ſecond, 2 ſeconds, and 3 ſeconds, the velocities generated will be then 6 inches, 6 × 2 inches, and 6 × 3 inches in a ſecond, reſpectively being in the ſame proportion with the times in which the given force acts.

In general, when the quantities of matter and forces are the ſame, the velocities generated will be as the times of motion, or when $\frac{M}{m} = 1$, and $\frac{Q}{q} = 1$, $\frac{V}{v} = \frac{T}{t}$

XLVII.

If any given quantity of matter is moved from quieſcence by different forces during a given time, the velocities acquired will be in the ſame proportion with the forces.

Let the maſs $= 64m$, and let the forces be m, $2m$, and $3m$ ſucceſſively: the velocities acquired in one ſecond of time, will be 6 inches, 12 inches, and 18 inches in a ſecond reſpectively, being in the ſame proportion with the forces. In general when $\frac{Q}{q} = 1$, and $\frac{T}{t} = 1$, $\frac{V}{v} = \frac{M}{m}$.

XLVIII.

If a given force impels different quantities of matter for the ſame time, the velocities generated will be inverſely as the quantities of matter.

If the given force is m, and the quantities of matter $64m$, and $32m$, the velocities acquired in two ſeconds will be 12 inches and 24 inches in a ſecond.

XLIX.

If a body is moved from quieſcence through the ſame ſpace by different forces, the velocities generated will be in a ſubduplicate ratio of the forces.

Let the maſs be $=64m$ and the force $=m$. The velocities acquired in deſcribing 12 inches will be 12 inches in a ſecond. Now let the maſs $=64m$, and the force $=4m$: the body in deſcribing 12 inches will acquire a velocity of 24 inches in a ſecond: the velocities acquired being in a ſubduplicate ratio of the forces. In general $\frac{V}{v}=\sqrt{\frac{S}{s}\times\frac{M}{m}\times\frac{q}{Q}}$; when $\frac{S}{s}=1$, and $\frac{Q}{q}=1$, $\frac{V}{v}=\sqrt{\frac{M}{m}}$.

L.

If a given quantity of matter is impelled from quieſcence, through different ſpaces by the action of the ſame force, the velocities generated will be in a ſubduplicate ratio of the ſpaces deſcribed.

When $\frac{M}{m}=1$, and $\frac{Q}{q}=1$, $\frac{V}{v}=\sqrt{\frac{S}{s}}$.

Force is always homogeneal with weight, and its quantity is eſtimated by comparing it with the force which the earth's gravity exerts upon ſome known or ſtandard ſubſtance.—Thus the action of any animal muſcle may be counterpoiſed by a weight: ſo may the elaſtic force of the air or that of a mechanical ſpring: and the quantity of the counterpoiſing weight will give the force of the muſcle or ſpring. The accelerating force is a number which expreſſes the proportion of the velocities generated in two quantities of matter by their reſpective impelling forces in the ſame time. All accelerating forces are referred to that of the earths gravity as a ſtandard; this force is ſuch as will generate a velocity of about $32\frac{1}{6}$ feet in a ſecond, in bodies which fall freely. And becauſe when any other conſtant forces are applied to accelerate a body, the velocities generated in a given time are as the forces directly and quantities of matter of inverſely by Art. 48 and Art. 49 it follows that the accelerating forces will obſerve the ſame proportion, being as the forces directly and the quantities of matter moved inverſely: let M and m repreſent the forces which impel the bodies Q and q: the proportion of the accelerating forces $F : f$ will be that of, $M \times q, : m \times Q$ or $\frac{F}{f} = \frac{M}{m} \times \frac{q}{Q}$.

When bodies are accelerated from quieſcence, the relation of the ſpaces deſcribed, velocities acquired, times of motion and accelerating forces will be expreſſed as follows.

$$\frac{S}{s} = \frac{F}{f} \times \frac{T^2}{t^2} = \frac{V}{v} \times \frac{T}{t} = \frac{V^2}{v^2} \times \frac{f}{F};$$

$$\frac{V}{v} = \sqrt{\frac{F}{b}} \times \sqrt{\frac{S}{s}} = \frac{S}{s} \times \frac{t}{T} = \frac{F}{f} \times \frac{T}{t};$$

$$\frac{T}{t} = \sqrt{\frac{S}{s}} \times \sqrt{\frac{\ }{F}} = \frac{S}{s} \times \frac{v}{V} = \frac{f}{F} \times \frac{V}{v}.$$

Theſe expreſſions are general and are applied to eſtimate the effects of any conſtant forces.

If f be put $= 1$, being the accelerating force of gravity, $t = 1$ ſecond, $s = 193$ inches $= l$; then $v = 2l$, and the preceding expreſſions become thoſe which follow, in which the force referred to as a ſtandard is that of the earth's gravity:

$S = l$

$$S = lFT^2 = \frac{V \times T}{2} = \frac{V^2}{4lF}.$$

$$V = \sqrt{4lFS} = \frac{2S}{T} = 2lFT.$$

$$T = \sqrt{\frac{S}{lF}} = \frac{2S}{V} = \frac{V}{2lF}.$$

Which will give the abſolute ſpaces deſcribed, times of deſcription, and velocities acquired by any conſtant accelerating forces, provided they are expreſſed in reference to that of gravity aſſumed as $= 1$, the velocity acquired being eſtimated by the ſpace that would be deſcribed uniformly in one ſecond of time.

LI.

If two bodies begin to deſcend from the higheſt point of an inclined plane at the ſame inſtant, one of them will fall through the perpendicular height, while the other deſcends on the plane to the interſection of a perpendicular drawn from the oppoſite angle.

In this caſe the ſpaces deſcribed are as the forces of acceleration, or $\frac{S}{s} = \frac{F}{f}$: therefore, becauſe in general $\frac{T}{t} = \sqrt{\frac{S}{s} \times \frac{f}{F}}$, the times of motion muſt be the ſame.

In this experiment each point of the deſcending body is ſuppoſed to move with the ſame velocity, being equal to that with which the centre of gravity moves; But if the body which deſcends along the plane ſhould roll, the times of motion will be different from thoſe which are expreſſed in the article.

If the body deſcends on wheels, the weight of which bears a very ſmall proportion to that of the body, by deſcending in this manner, it will be accelerated nearly in the ſame degree as if it ſlided down the plane without friction. This remark is applicable to the next article.

LII.

If bodies deſcend along inclined planes which have the ſame height, the times of deſcent will be as the lengths of the planes.

In general $\frac{S}{s} = \frac{F}{f} \times \frac{T^2}{t^2}$; but in this article $\frac{f}{F} = \frac{S}{s}$ which being ſubſtituted, we have $\frac{S^2}{s^2} = \frac{T^2}{t^2}$ or $\frac{S}{s} = \frac{T}{t}$: that is, the lengths of the planes are in the ſame proportion with the times of motion.

LIII.

If equal quantities of matter are projected in free ſpace with any given velocity, and are reſiſted by different but invariable forces, the ſpaces deſcribed before the whole motion is deſtroyed will be inverſely as thoſe reſiſting forces.

Let a maſs of 61 *m* be projected with a velocity of 18 inches in a ſecond, and let it be reſiſted ſucceſſively by the forces *m*, 2*m*, and 3*m*, the ſpaces deſcribed before the motion of the body is deſtroyed will be $\frac{25.6}{1}$ $\frac{25.6}{2}$ and $\frac{25.6}{3}$ theſe ſpaces being in the inverſe proportion of the reſiſting forces.

The properties of acceleration are applied to thoſe of retardation by changing the velocity acquired for the velocity of projection or uniform motion when the reſiſtance is applied. Moreover the moving force muſt be changed into reſiſting force, and the accelerating force, which was proportional to the moving force directly and quantity of matter inverſely, or to the velocity generated in any given time, when theſe propoſitions are applied to retardation, will become the retarding force, which is proportional to the reſiſting force divided by the quantity of matter, or to the velocity which is deſtroyed in a given time.

Becauſe $\frac{V^2}{v^2} = \frac{S}{s} \times \frac{F}{f}$: when $V = v$ as in the experiment $\frac{S}{s} = \frac{f}{F}$: or the ſpaces are in the inverſe proportion of the retarding forces.

LIV.

If any body moving in free ſpace uniformly is reſiſted by a conſtant force, for any given time leſs than that in which the whole motion would be deſtroyed, the ſpace deſcribed will be the difference between the ſpace which meaſures the initial velocity of motion multiplied into a number expreſſing the given time, and that ſpace which the body would deſcribe if accelerated during the given time from quieſcence by a force equal to that of reſiſtance.

Theſe properties and others relating to the ſubject, admit of experimental illuſtration with the greateſt exactneſs.

In general the ſpace deſcribed by a body conſtantly retarded by the force F in the time T, eſtimated from the firſt moment of reſiſtance, will be $= VT - lFT^2$.

Let a maſs $= 63\frac{1}{2}m$ be projected with a velocity of 11.87 inches in a ſecond, if it is reſiſted by a force $= \frac{1}{2}m$

it will deſcribe 21.95 inches in 3 ſeconds. Here $V = 11.87$, $T = 3$, $F = \frac{1}{127}$, $l = 193$ inches: and $S = VT - lFT^2 = 35.61 - 13.66 = 21.95$ inches.

LV.

If ſpherical bodies having deſcended from reſt through different ſpaces, impinge on ſnow, clay, or other ſimilar given ſubſtance of uniform texture, the depths to which they ſink will be in the joint proportion of the heights they fall from, their diameters and ſpecific gravities.

The forces with which ſubſtances ſuch as ſnow, clay, &c. reſiſt the progreſs of ſpherical bodies that impinge on them are nearly uniform, ſetting aſide what irregularity may ariſe from the figure of the impinging bodies: it is certain that during the gradual motion of a ſphere impinging on any ſubſtance to a depth equal to its radius, the force of reſiſtance will vary conſiderably; but as the whole depths are underſtood to be much greater than the radii of the ſpheres, and the reſiſtance is uniform after the ſphere has ſunk to a depth equal to radius, it is conſidered wholly ſo without creating any objectionable error in a ſubject, where mathematical preciſion is not neceſſary.

This being granted, the reſiſtances oppoſed by the ſame ſubſtance will not depend on the velocities of impact or on the denſities of the ſpheres, but on their diameters only, being in a duplicate ratio of them: let $\frac{H}{h}$ repreſent the ratio of the altitudes from which the ſpheres deſcend before they impinge on the ſurface of the clay, or other ſubſtance: let $\frac{D}{d}$ be the ratio of the diameters of the ſpheres $\frac{N}{n}$ that of their ſpecific gravities;

then the general notation being obſerved, we have, $\frac{S}{s} = \frac{V^2}{v^2} \times \frac{f}{F}$ or $\frac{S}{s} = \frac{H}{h} \times \frac{m}{M} \times \frac{Q}{q}$: but $\frac{m}{M} = \frac{d^2}{D^2}$, and $\frac{Q}{q} = \frac{N}{n} \times \frac{D^3}{d^3}$: which quantities being ſubſtituted, it appears that $\frac{S}{s} = \frac{H}{h} \times \frac{N}{n} \times \frac{D}{d}$; that is, the ratio of of the whole ſpaces or depths to which the ſpheres penetrate, will be equal to the ratio of the altitudes from which they have deſcended, added to the ratio of the ſpecific gravities, and that of their diameters. In this caſe the ſpheres are ſuppoſed to impinge on the ſame ſubſtance: if the ſubſtances are different, the ratio of their abſolute reſiſtances ariſing from their different textures muſt be known: this ratio will be known from the depths to which two equal ſpheres of the ſame denſity and impinging with the ſame velocity ſink into the ſubſtances reſpectively being the inverſe ratio of thoſe depths: if this ratio of the abſolute reſiſtances be that of R to r; then $\frac{S}{s} = \frac{H}{h} \times \frac{N}{n} \times \frac{D}{d} \times \frac{r}{R}$.

To this expreſſion may be referred the known experiment by which it is ſhewn that if ſpheres of equal diameter, having fallen from heights which are reciprocally as their ſpecific gravities impinge on the ſurface of clay, the depths to which they ſink will be equal: for in the value $\frac{S}{s} = \frac{H}{h} \times \frac{N}{n} \times \frac{D}{d} \times \frac{r}{R}$, $\frac{r}{R} = 1 = \frac{D}{d}$, and $\frac{H}{h} = \frac{n}{N}$ or $\frac{H}{h} \times \frac{N}{n} = 1$, by the conſtruction of the experiment; therefore $\frac{S}{s} = 1$ or the depths to which the ſpheres ſink are equal.

The preceding articles relative to the motion of bodies gradually accelerated and retarded are placed before thoſe, by which the impact of bodies is determined, becauſe the laws of colliſion ſeem to depend entirely on thoſe of gradual acceleration and retardation.

ON THE COMMUNICATION OF MOTION BY COLLISION.

LVI.

Action and reaction are equal and in contrary directions.

This propoſition is aſſumed as an axiom in Statics and Mechanics, and admits of very ſatisfactory illuſtration from experiments.

When a force of any kind is counterpoiſed, whether it be by the ſtrength of animal muſcles, the preſſure of a fluid, the elaſticity of a ſpring &c. each of theſe oppoſed forces will be equal to ſome given weight, which, if it be ſubſtituted ſo as to act inſtead of the force, will preſerve the equilibrium unaltered. Even an immoveable obſtacle exerts a definite force equal and contrary to that which is applied in any manner or degree to alter its poſition: for ſuppoſe the obſtacle to be taken away, a force equal and contrary to that by which it was before urged muſt be ſubſtituted in its place to preſerve the equilibrium.

If a body impinges againſt another body moving in the direction of a line which joins their centres of gravity, the two bodies mutually act on each other in any given inſtant of their colliſion with equal forces, which equal forces will always have the effect of retarding the ſtriking body, and of accelerating the body ſtruck if it moves in the ſame direction with the ſtriking body, and of retarding it, if it moves in the contrary direction.

This mutual action and reaction of bodies may be illuſtrated by ſuppoſing a ſpring in the form of an Helix the axis of which coincides with the direction of motion to be interpoſed between the two bodies; then, in whatever degree the ſpring is compreſſed, it will exert the ſame elaſtic force on each of the bodies (The ſprings inertia not being conſidered). When the ſpring firſt begins to be compreſſed, the force by which it retards the ſtriking body and accelerates the body ſtruck is the leaſt of all: it afterwards will continually increaſe during each ſucceſſive inſtant, till its compreſſion is the greateſt poſſible. The intenſity of this force may vary according to any imaginable law, but whatever be the variation, it will act equally on both bodies during the ſame inſtant.

When the two bodies have acquired a common velocity, the ſpring can ſuffer no further compreſſion: and may then begin to reſtore itſelf with various degrees of force from 0 to the force by which it was compreſſed. When the reſtitutive force is $= 0$, the caſe anſwers to that of nonelaſtic bodies; when the reſtitutive force is equal to that of compreſſion, the action of the ſpring will be ſimilar to that of perfectly elaſtic bodies.

LVII.

If a nonelaſtic body A, moving with the velocity a, impinges on a body B, moving with the velocity b, and the direction of the ſtroke is in a line which joins the centres of gravity, the following proportion will determine the velocity communicated to B: as $A + B : A :: a - b :$ to the velocity acquired by F.

Motion cannot be communicated to any body inſtantaneouſly, but muſt be produced by gradual acceleration, it not being conceivable that any really exiſting body ſhould paſs from quieſcence into finite motion, or from one degree of finite motion to another, without having poſſeſſed all the intermediate degrees of velocity.

Suppoſe a ſpherical body to impinge on another body of the ſame form, moving in the ſame direction with the line which joins their centres, and let both of them be perfectly nonelaſtic. When the ſurfaces of theſe ſpheres are juſt in contact, the diſtance of their centres will be juſt half the ſum of their diameters; but as the figures are gradually changed by the impact; their centres will become nearer than before; during the time of their approach, the centre of the ſtriking ball will move with a greater velocity than that of the ball ſtruck; and the reſiſtance which the ſpheres oppoſe to the change of their figures, will act equally on both ſpheres in contrary directions; that is, the force by which the ſtriking body is reſiſted, will urge forward the body ſtruck, which therefore will be gradually accelerated, and the ſtriking ball retarded, until the centres are at their leaſt diſtance, the change of the bodies figures being then the greateſt, at which inſtant all acceleration of the ball ſtruck and retardation of the ſtriking ball ceaſe, and the two centres begin to go on with a common velocity.

This reaſoning concerning the colliſion of bodies is equally applicable to the caſe in which both bodies are nonelaſtic, as when one is perfectly nonelaſtic and the other perfectly hard. It is likewiſe equally applicable to the colliſion of bodies which are elaſtic in any degree, ſo far as regards the velocity communicated to the body ſtruck during that ſmall but finite portion of time in which the figures of the ſpheres receive their greateſt change, their centres then beginning to go on with a common velocity in bodies of every kind of texture.

In elaſtic bodies the power which reſtores the changed figures will depend upon the textures of the bodies, and may be in any degree variable from being nothing (this anſwering to the caſe of nonelaſtic bodies) to its being equal to the force by which this change was occaſioned. Bodies which poſſeſs this reſtitutive power in ſo great a degree are termed perfectly elaſtic: but whatever be the intenſity of the elaſtic power it will always add to the velocity acquired by the body ſtruck, during the change of figures, and will diminiſh the velocity of the ſtriking body.

The common velocity is next to be aſcertained, the weights and velocities of the bodies before the impact being given.

The time in which the additional velocity is acquired by the body ſtruck, that is, in which the figures of the bodies receive the greateſt change being finite, may be divided into moments or evaneſcent inſtants, and the force by which one or both bodies reſiſt the change of their figure cauſed by the impact, during one of theſe inſtants, acts equally and in oppoſite directions on the two bodies (Art. LIV.) notwithſtanding in different inſtants this force may in any degree vary; but the force being in a given inſtant the ſame in reſpect of both bodies, the velocity generated in the body ſtruck and that loſt by the ſtriking body will be in the inverſe proportion of the quantities of matter contained in the bodies: that is the velocity communicated to B will be to the velocity loſt by A, as the latter body is to the former. And this proportion being the ſame through all the ſucceſſive inſtants till the greateſt change of figures takes place, whatever be the magnitude of the force which acts in each inſtant, it will follow that the whole velocity communicated to B, will be to the velocity loſt by A as A to B. let y be the velocity communicated to B, and let x be that which is loſt by A: then by what has preceded, as $x : y :: B : A$, and $x = \frac{By}{A}$; therefore the common velocity of A and B, when their change of figures in the greateſt, is

$a - \frac{By}{A} = b + y$; from whence is obtained $y \times \overline{A+B} = A \times \overline{a-b}$ which equation is reſolved into this analogy.

As $A+B : A :: a-b : y =$ the velocity communicated to B.

It is evident from the notation that $a-x=b-y$ are each equal to the common velocity of A and B.

LVI.

An ivory ball impinging againſt a quieſcent ivory ball of twice its weight in the direction of a line which joins their centres, communicates to the ball ſtruck, nearly two thirds of the velocity with which the impact was made.

If the balls are perfectly elaſtic, the general rule for determining the effects of colliſion is this: as the ſum of the bodies is to twice the ſtriking body, ſo is the difference of the velocities before the impact (eſtimated in the ſame direction) to the velocity acquired by the body ſtruck.

The time in which the effects of colliſion of elaſtic bodies are produced, ſhould be conſidered as being divided into two portions; the firſt is that in which the figures of the bodies receive their greateſt change, at the end of which time the bodies begin to go on with the ſame velocity; the other portion of time that in which the changed figures are reſtored.

Let the ſtriking ball mentioned in the article prefixed be A, moving with the velocity a: let B be the ball ſtruck moving with the velocity b: alſo let x be the velocity loſt by A, and y that which is gained by B during the firſt portion of time, at the end of which the velocities of A and B are equal.

The two portions of time above referred to, may be either equal or unequal; but whatever be the proportion between them, they may be divided into an equal number of inſtants.

Let the elaſtic force be ſuch that the velocity acquired by the body ſtruck in any given inſtant in conſequence of the elaſticity may be to the velocity which was before acquired in the inſtant correſponding as $m : 1$, m being aſſumed as a meaſure of elaſtic force.

The whole velocity acquired by the body ſtruck from the elaſtic force will be to that which was before acquired, in the ſame proportion of $m : 1$. this will give the entire velocity of B after the impact $= b + y + my$, and by ſimilar reaſoning, the velocity of A after the impact $= a - x - mx$; or of $m + 1$ be put $= n$, the velocity of A after the ſtroke $= a - nx$, and that of $B = b + ny$; but $x = \frac{By}{A}$ in general: wherefore the velocity of A after the ſtroke $= \frac{Aa - Bny}{A}$ and that of $B = b + ny$.

Multiply each body into its velocity after the impact; the ſum of the products will be $Aa - Bny + Bb + Bny = Aa + Bb$ the ſame as the ſum of the prodncts formed by multiplying each body into its velocity before the ſtroke.

Let the elaſtic force be equal to the force which cauſes the change of figures; then $m = 1$ and $n = 2$: the velocity of A after the ſtroke $= \frac{Aa - 2By}{A}$ or becauſe $y = \frac{\overline{a - b} \times A}{A + B}$ by ſubſtituting this value of y, the velocity of A after the impact will be $= \frac{Aa - Ba + 2Bb}{A + B}$; and the velocity of B after the ſtroke $= b + ny = \frac{Bb + 2Aa - Ab}{A + B}$

The velocity gained by B is $ny = \frac{A\times\overline{m+1}\times\overline{a-b}}{A+B}.$

for ſince $m=1$ the velocity gained by $B = \frac{2A\times\overline{a-b}}{A+B}$

which is reſolved into this analogy: as $A+B : 2A :: a-b$ to the velocity acquired by B.

Every thing remaining becauſe $n=2$, the velocity of A after the ſtroke $=a-2x$, and that of B will be $b+2y$; and the difference between the velocities of A and $B=a-b-2x-2y$, or becauſe $a-x=b+y$ or $x+y=a-b$ in general, by ſubſtitution, the difference of the velocities after the impact will be $a-b-2a+2b=b-a$, the ſame as the difference of the velocities before the ſtroke only negative in reſpect of it.

From this property, the propoſition ſo much inſiſted on by Bernouilli may be deduced: this is, that in the colliſion of perfectly elaſtic bodies, the ſum of the products formed by multiplying each body into the ſquare of its velocity, is not alltered by the impact.

His demonſtration is this: the other notation and conditions remaining, let p be the velocity of A, and q that of B, after the ſtroke: then by a previous demonſtration $Aa+Bb=Ap+Bq$ (which has likewiſe been demonſtrated an Art. LIII.) or $Aa-Ap=Bq-Bb$: It has been likewiſe ſhewn that $a-b=q-p$, or $a+p=q+b$ in the colliſion perfectly elaſtic bodies (ſupra): wherefore by multiplying $Aa-Ap$ into $a+p$, and $Bq-Bb$ into $q+b$ there will ariſe this eqnation:

$$Aa^2-Ap^2=Bq^2-Bb^2 \text{ or}$$

$$Aa^2+Bb^2=Ap^2+Bq^2$$ which is the equality that was affirmed. Vid. Bernouilli, Vol. IV. Diſcours ſur le Mouvement. A geometrical demonſtration is alſo given by Maclaurin of the ſame propoſition.

If it is enquired what muſt be the force of elaſticity that the ſums of the products formed by multiplying each body into any power of the velocity which is denoted by the general index r may not be altered by the impact, the ſolution will be obtained from the general expreſſions for the velocities of A and B after the impact.

The former notation remaining, ſince the velocity of A after the ſtroke is $a - \frac{Bny}{A}$ and that of $B = b + ny$, the conditions of the problem will give this equation: $\frac{\overline{Aa - Bny}^{\,r}}{A^{r-1}} + B \times \overline{b + ny}^{\,r} = Aa^r + Bb^r$; in which quantities $n = m + 1$, and $y = \frac{A \times \overline{a - b}}{A + B}$, m being the elaſtic force when that which cauſes the change of figures in the correſponding inſtant (whether unequal or equal to the other) is $= 1$. The ſolution of the equation, n being conſidered as the unknown quantity, will give the value of $m = n - 1$. Thus if r is aſſumed $= 2$ which is the caſe in the demonſtration of Bernouilli juſt referred to, by ſubſtituting 2 for r in the general equation, n will come out $= 2$, and conſequently $m = 1$: that is, the force of elaſticity muſt be equal to the force which cauſed the change of figures, ſo that the ſums of the products formed by multiplying each body into the ſquare of its velocity before and after the ſtroke may be equal. In this caſe the bodies are ſaid to be perfectly elaſtic. If r be $= 3$, the value of m will depend on thoſe of A and B and of a and b: let $A = 2$, and $B = 1$, and ſuppoſe $b = 0$; in which caſe, B is quieſcent before the ſtroke: then by ſubſtituting 3 for r, 2 for A, &c. in the general equation, it will appear that $n = 1.854$ and m the elaſtic force required $= .854$. If therefore the elaſtic force be to that which cauſes the change of figure as .854 to 1, the ſum of the products formed by multiplying each body into the cube of its velocity will be the ſame before and after the impact.

LVII.

Let the centres of any number of contiguous Ivory balls equal to each other be placed in a right line; and let the firſt impinge againſt ſecond in the direction of that right line: it is obſerved that the ball which is moſt remote from the ſtroke is impelled with a velocity nearly equal to the velocity of impact while the ſtriking ball with all the others intermediately between that and the remote ball remain almoſt quieſcent.

If the balls were perfectly elaſtic, they would be perfectly quieſcent, and the velocity of the remote ball exactly equal to that of the ſtriking ball.

The difference ariſing from the imperfect elaſticity is very ſmall.

LVIII.

If an elaſtic ball impinges directly againſt a quieſcent elaſtic ball of greater weight, the ſtriking ball will be reflected; if a heavier ball ſtrikes againſt a lighter one at reſt, both the balls after the impact will go on in the direction of the ſtroke.

In the notes concerning the colliſion of bodies, they have been ſuppoſed to move in the ſame direction, that repetition might be avoided, and that the reaſoning on the ſubject might be more obvious. The principles deduced are equally applicable to the caſe in which a body impinges againſt another moving in the contrary direction, only by changing the ſign of the quantity which is put to denote the velocity of the body ſtruck before the impact.

In whatever degree the hardneſs of perfectly elaſtic bodies may differ, the effects of their impact will be the ſame, their weights and velocities before the ſtroke being given: for the figures of the two bodies muſt continually change, till they have acquired a common velocity, which depends only on the weights of the bodies and their velocities before the impact, and is determined by the rules for the colliſion of nonelaſtic bodies: moreover during the reſtoration of the changed figures, how ſmall or great-ſoever may have been that change, the elaſtic force will cauſe an additional velocity in the ball ſtruck, equal to that before generated during the change of the figures.

It follows alſo from what has preceded, that the effects of the impact will be the ſame, whether both bodies be perfectly elaſtic, or one perfectly elaſtic, and the other perfectly hard, every thing elſe being the ſame: for, ſuppoſing one of them to be perfectly hard, the figure of the elaſtic body muſt change till both bodies move with a common velocity, which depends on the weights and velocities before the impact only, and will be the ſame as if the bodies were nonelaſtic: the reſtoration of the figure will, in this as in the former caſe, cauſe an increaſe of the velocity in the body ſtruck, equal to that which was before communicated.

ON

ON THE ROTATION OF BODIES ROUND FIXED AXES.

LIX.

The particles of matter which compoſe any body or ſyſtem of bodies revolving round a fix'd axis reſiſt by their inertia the communication of motion to any given point with forces which are as the particles themſelves and the ſquares of their diſtances from the common axis jointly.

Suppoſe a force p, to be applied at any point e ſituated at the diſtance d from the axis in order to communicate motion to a ſyſtem which conſiſts of the particles A, B, C revolving at the diſtances a, b, c reſpectively from the common axis,

Let z be a quantity of matter which if concentrated in e will have the ſame effect in reſiſting the communication of motion to the point e by its inertia, when the particle A is removed, as the particle A revolving at the diſtance a :. then by the propoſition contained in the article $A \times a^2 = z \times d^2$ or $z = \frac{A \times a^2}{d^2}$; from whence it appears that the inertia of the particle A revolving at the diſtance a, when referred to a point at the diſtance d is equivalent to the inertia of the maſs $\frac{A \times a^2}{d^2}$ concentrated in a point revolving at the diſtance d from the axis.

For the ſame reaſons the inertia of the point B revolving at the diſtance b, is equal to that of $\frac{B \times b^2}{d^2}$ revolving at the diſtance d: and the inertia of C revolving at the diſtance c is equal to that of $\frac{C \times c^2}{d^2}$ revolving at the diſtance d.

LX.

If a body revolves round a fix'd axis by the action of a force p applied at the diſtance d to turn the ſyſtem, the force which accelerates the point to which p is applied, will be a fraction, the numerator of which is p, and the denominator the ſum of all the equivalent maſſes found according to the rule contained in the laſt article.

The notation of the laſt article remaining, the moving force will be p, and the maſs moved will be $\frac{Aa^2}{d^2} + \frac{Bb^2}{d^2} + \frac{Cc^2}{d^2}$: conſequently the force which accelerates the point to which p is applied will be $\frac{pd^2}{Aa^2 + Bb^2 + Cc^2}$.

In this value the inertia of p is aſſumed $= 0$. If p poſſeſſes inertia, the quantity pd^2 muſt be added to the denominator.

The centre of gyration is a point in which if all the matter contained in a revolving ſyſtem were collected, any point to which a given force is applied to communicate motion, would be accelerated in the ſame manner as when the parts of the ſyſtem revolve in their reſpective places: and conſequently the angular velocity generated in a given time the ſame in both caſes.

LXI.

If the ſum of the products formed by multiplying each particle of a ſyſtem into the ſquare of its diſtance from the axis of motion be divided by the weight of the whole, the ſquare root of the quotient will be the diſtance of the centre of gyration from the axis of motion.

Let A, B, C be the particles which compoſe the ſyſtem; and let the diſtances at which they revolve be a, b, c reſpectively: if a force without inertia be applied at the diſtance d to turn the ſyſtem, the force which accelerates the point to which p is applied will be $\frac{pd^2}{Aa^2+Bb^2+Cc^2}$: now let A+B+C be collected into a point at ſome unknown diſtance r, ſo that the force which accelerates the point to which p is applied may be the ſame as before: this will give $\frac{pd^2}{Aa^2 + Bb^2 + Cc^2}$ $= \frac{pd^2}{\overline{A+B+C}\times r^2}$ and $r = \sqrt{\frac{Aa^2+Bb^2+Cc^2}{A+B+C}}$, the diſtance of the centre of gyration from the axis of motion.

When the axis of motion paſſes through the centre of gravity, the centre above determined is called the principal centre of gyration.

LXII.

LXII.

In any ſyſtem which revolves round a fix'd axis, having given the diſtance of the centre of gyration from the axis $=r$, the weight of the ſyſtem $=w$, the conſtant moving force p, and the diſtance from the axis at which it is applied $=d$, it is required to determine the angular velocity generated in a given time.

Suppoſe the axis to paſs through the centre of gravity of the ſyſtem, and let the moving force be a weight p applied to act on the ſyſtem by means of a line wound round a cylinder which has the ſame axis with that of the ſyſtem, the radius of the cylinder being d: then the moving force to impel p or the point to which it is applied, is p: the inertia or maſs moved when referred to p is $p+\frac{wr^2}{d^2}$; conſequently the force which acclerates the deſcent of $p=\frac{pd^2}{pd^2+wr^2}$: if $l=16\frac{1}{6}$ feet, and $c=3.1416$ the angular velocity generated in t ſeconds $=\frac{ltpd}{cd^2p+cr^2w}$ revolutions or parts of a revolution in a ſecond.

In bodies of uniform denſity that admit of geometrical menſuration the centre of gyration may be aſcertained from theory: but in bodies, the denſities or dimenſions of which are irregular, ſome experiment will be neceſſary for the determination of this point.

LXIII.

The notation of the laſt article remaining, ſuppoſe r unknown; and let the weight p deſcend freely from reſt: having the ſpace deſcribed by it from reſt in any given time, the centre of gyration will be determined.

Suppoſe s to be the ſpace deſcribed in t ſeconds: becauſe $\frac{pd^2}{d^2p + wr^2}$ is the accelerating force, the ſpace deſcribed by p in t ſeconds will be $\frac{lt^2pd^2}{d^2p + wr^2}$: which is equal to s by the ſuppoſition: reſolving the equation $s = \frac{lpt^2d^2}{pd^2 + wr^2}$, r will be found $= \sqrt{\frac{lpt^2d^2 - spd^2}{sw}}$.

XLIV.

Any body ſliding down an inclined plane without friction, will deſcribe a ſpace which is to the ſpace deſcribed by a ſphere that rolls down the ſame inclined plane during the ſame time, as ſeven to five, the deſcents being eſtimated from quieſcence.

The abſolute force which impels the ſphere's centre of gravity is the weight of the ſphere w, multiplied into the height of the plane h, and divided by the length l: the maſs moved is that of the ſphere added to the ſame maſs multiplied into the ſquare of the diſtance of the principal centre of gyration from the ſphere's centre $\frac{2d^2}{5}$ and divided by the ſquare of the ſphere's radius d^2; this gives the force which accelerates the deſcent of the ſphere $=\frac{wb}{l}\times\frac{1}{w+\frac{2w}{5}}=\frac{5b}{7l}$. When any body ſlides down the ſame inclined plane, the accelerating force is $\frac{b}{l}$ which is to the former force as 7 : 5 : and conſequently the ſpaces deſcribed in the ſame time will be in the proportion of 7 to 5.

The centre of oſcillation is a point in which if all the matter in a ſyſtem were collected, any force applied at this centre will generate the ſame angular velocity in a given time, as the ſame force when applied at the centre of gravity; the parts of the ſyſtem revolving in their reſpective places.

LXV.

If the ſum of the products formed by multiplying each particle of a revolving ſyſtem into the ſquare of its diſtance from the axis of motion be divided by the weight of the whole × into the diſtance of the centre of gravity from the axis of motion, the quotient will be the diſtance of the centre of oſcillation from the ſame axis.

Let the diſtance of the centre of oſcillation from the axis be x, that of the centre of gravity g: then if a force p is applied at the diſtance g, the force which accelerates the centre of gravity $= \frac{g^2p}{Aa^2+Bb^2+Cc^2}$; and the force which accelerates the centre of oſcillation $= \frac{gxp}{Aa^2+Bb^2+Cc^2}$: now let the ſum of the particles be collected into the centre of oſcillation, and let the force p be applied there; the force which accelerates that point will be $= \frac{p}{A+B+C}$: but becauſe the angular velocities are to be equal in both caſes, it follows that any given point, for example that to which the force p is now applied, muſt be acted on by equal accelerating forces: wherefore it will follow that $\frac{gxp}{Aa^2+Bb^2+Cc^2} = \frac{p}{A+B+C}$ and $x = \frac{Aa^2+Bb^2+Cc^2}{g \times \overline{A+B+C}}$ the diſtance of the centre of oſcillation from the axis of motion.

The diſtance of the centre of gyration from the axis of motion is a mean proportional between the diſtances of the centres of oſcillation and gravity from the ſame axis.

The centre of percuſſion is the ſame point with the centre of oſcillation.

Any body or ſyſtem of bodies which is ſuſpended ſo as to vibrate freely on an axis of ſuſpenſion is called a pendulum.

When a pendulum is accelerated in its vibratory motion, the motive force is that of its weight multiplied into the ſine of the angular diſtance of the centre of gravity from the loweſt point and divided by radius; this force acts at the centre of gravity: if a point be imagined in which all the matter in the ſyſtem being concentrated the ſame angular velocity will be generated in a given time as when the parts of the ſyſtem revolve in their reſpective places, this point will be the centre of oſcillation which has been deſcribed in the preceding article; and conſequently a pendulum will vibrate in the ſame time through a given arc when the maſs is collected into the centre of oſcillation as when the parts vibrate at their reſpective places.

The length of a pendulum is equal to the diſtance between the axis of ſuſpenſion and centre of oſcillation.

LXVI.

The time in which a pendulum performs one vibration in the leaſt arc of a circle is to the time of a body's deſcribing from reſt half the pendulum's length by falling freely, as the circumference of a circle to its diameter.

Let $l = 193$ inches, $L =$ the length of the pendulum in inches, $T =$ the time of one vibration, $c = 3.1416$,

then $T = \frac{c \times \sqrt{L}}{\sqrt{2l}}$ ſeconds.

If in any time T, the error of a pendulum clock is $\pm \dot{T}$ when the length of the pendulum is L, the due correction of the length is $\pm L \times \frac{2\dot{T}}{T}$; $\dot{T}$ being conſidered as a fluxional quantity, or ſmall finite variation.

LXVII.

If a pendulum vibrates in very ſmall circular arcs, the times of vibration will be equal whatever be the proportion of the arcs.

The ſorces which accelerate the centre of gravity or oſcillation being as the ſines of the angular diſtances from the loweſt point, theſe forces will be as the arcs themſelves when they are very ſmall: but whenever the forces are as the ſpaces to be deſcribed from reſt, the times of deſcription are equal.

LXVIII.

Pendulums which are of the ſame length vibrate in the ſame time whatever be the proportion of their weights.

This follows from the property of gravity, which is always proportional to the quantity of matter or to its inertia.

When the vibrations of pendulums are compared, it is always underſtood that the pendulums deſcribe either ſimilar finite arcs, or arcs of evaneſcent magnitude, unleſs the contrary is mentioned.

LXIX.

If an axis of motion be made to paſs through different points of a body or ſyſtem of bodies, perpendicular to any given plane paſſing through the body or ſyſtem, the different pendulums will be ſo conſtituted that the diſtances between the centre of gravity and the axes of motion, will be inverſely proportional to the diſtances between the centres of gravity and oſcillation.

The diſtance between the centre of gravity and principal centre of gyration is a mean proportional between the diſtance of the centre of gravity from the axis of mo-

motion, and the diſtance between the centres of oſcillation and gravity; the diſtance of the principal centre of gyration from the centre of gravity is always the ſame in the vibrating ſyſtem deſcribed in the article. From whence the truth of the propoſition evidently appears.

LXX.

The time in which a given body or ſyſtem of bodies vibrates as a pendulum round different axes which are all perpendicular to the ſame plane drawn through the body will be the leaſt poſſible when the axis of motion paſſes through the principal centre of gyration.

Let the diſtances of the principal centre of gyration, the axis of ſuſpenſion, and centre of oſcillation from the centre of gravity reſpectively be r, x, and y: then $x : r :: r : y$; wherefore by the laſt prop. of the Vth Euclid $x+y$ is always greater than $2r$, unleſs $x=y=r$ in which extreme caſe r being a given quantity $x+y$ muſt be the leaſt poſſible; and $x+y$ is the length of the pendulum by the ſuppoſition, which in this caſe will be $=2r$.

LXXI.

The times in which different pendulums vibrate in ſimilar arcs of circles, are in a ſubduplicate ratio of their lengths directly and a ſubduplicate ratio of the abſolute forces of gravity inverſely.

Let the ſimilar arcs be divided into an equal number of ſimilar evaneſcent arcs: then the times of deſcribing any two of theſe correſponding evaneſcent arcs will be as thoſe arcs directly and the velocities of the pendulums deſcribing them inverſely; that is from ſimilitude of figures, as the lengths of the pendulums directly and the velocities inverſely. But the velocities are in a ſubduplicate ratio of the heights fallen from and the forces of gravity jointly, or from the ſimilitude of figures, in a ſubduplicate ratio of the lengths of the pendulums and the forces of gravity; wherefore the times of deſcribing theſe evaneſcent arcs will be as the ſquare roots of the lengths directly, and the ſquare roots of the forces of gravity inverſely. And by the ſame reaſoning the times of deſcribing all the other ſimilar correſponding evaneſcent arcs, and conſequently the times of deſcribing entire arcs, will be in the ſame proportion.

LXXII.

At any part of the arc deſcribed by the centre of oſcillation, its velocity is equal to that which a body would acquire by falling freely from reſt through a ſpace equal to the perpendicular height which the centre of oſcillation has deſcended through, eſtimated from the higheſt point of the vibration.

LXXIII.

When a pendulum is at reſt, if a body impinges in an horizontal direction on it, the ſame velocity will be communicated to the point of impact, as if the maſs of the pendulum were removed, and inſtead of it an equivalent maſs were concentrated in the point of impact, the quantity of the equivalent maſs being to that of the pendulum,

lum, in a duplicate ratio of the diſtances between the centre of gyration, and the point of impact from the axis of motion.

Let the maſs of the pendulum be w, the diſtance of impact from the axis $= d$, that of the centre of gyration $= r$; the equivalent maſs $= q$, the impinging body p: the velocity of impact V: the velocity communicated to the point of impact $= v$: then $q = w \times \frac{r^2}{d^2}$; and by the laws of colliſion, if the bodies are nonelaſtic, $p + w \times \frac{r^2}{d^2} : p ::$ as $V : v$; wherefore $v = \frac{Vpd^2}{d^2p + wr^2}$ or $V = \frac{w \times \overline{d^2p + wr^2}}{pd^2}$ which is Mr. Robin's theorem for eſtimating the velocity of a bullet that has communicated a velocity v to that point of the pendulum on which it impinged; the quantity v being determined from the arc A° which the pendulum has deſcribed by the force of the impact. If a be put for the verſed ſine of A° to radius d, and the diſtance of the centre of oſcillation from the axis of motion $= s$, then $v = \sqrt{\frac{4lda}{s}}$; l being $= 193$ inches.

LXXIV.

If a given body or ſyſtem of bodies vibrates round different axes ſituated at the ſame diſtance from the centre of gravity, the times in which the pendulums vibrate will be equal, provided all the axes are perpendicular to any given plane drawn through the body.

The diſtance between the centre of gravity and axis of motion is the ſame in all the pendulums, and the diſtance of the principal centre of gyration from the centre of gravity is a given quantity: wherefore the third proportionals to the former two quantities, that is, the diſtances between the centre of gravity and centres of oſcillation, muſt be equal in all the pendulums, which therefore will be all of equal lengths.

LXXV.

A pendulum which vibrates in the arc of a cycloid deſcribes the greateſt and leaſt entire vibrations in the ſame time.

This property is demonſtrated only on a ſuppoſition that the whole maſs of the pendulum is concentrated in a point; but this cannot take place in any really vibrating body: and when the pendulum is of finite magnitude, there is no point given in poſition which determines the length of the pendulum; on the contrary the centre of oſcillation will not occupy the ſame place in the given body when deſcribing different parts of the track it moves through; but will continually be moved in reſpect of the pendulum itſelf during its vibration. This circumſtance has prevented any general determination of the time of vibration in a cycloidal arc except in the imaginary caſe before referred to.

There are many other obſtacles which concur in rendering the application of this curve to the vibration of pendulums deſigned for the meaſures of time, the ſources of errors far greater than thoſe which by its peculiar property it is intended to obviate. It is now wholly diſuſed in practice.

LXXVI.

Bodies projected near the earth's ſurface in any direction not perpendicular to the horizon, deſcribe a parabola; the air's reſiſtance not being conſidered.

Very denſe bodies moving with ſmall velocities deſcribe the parabolic track ſo nearly, that any deviation from it is ſcarcely diſcoverable; but when very rare bodies move with velocities however ſmall, or denſe bodies with very great velocities ſuch as thoſe which are communicated by gunpowder to cannon and other bullets, the reſiſtance of the air will cauſe the body projected to deſcribe a path altogether different from a parabola. This may be eaſily imagined when it is found that the reſiſtance of the air to a cannon ball of 24 pound weight, moving with its initial velocity from a full charge of powder, is between 500 and 600 pounds; which reſiſtance is above 20 times the ball's weight.

LXXVII.

If the velocity of projection is given, the horizontal range is the greateſt when the angle of elevation is 45°.

This is true only on a ſuppoſition that the air's reſiſtance is of no ſenſible effect: but military projectiles are ſo much influenced by the air's reſiſtance, that the greateſt horizontal range will not correſpond to an angle of elevation of 45°, but to an angle much leſs, depending on the initial velocity, denſity, and diameter of the ball.

HYDROSTATICS.

Article I.

Let any light veſſel filled with water be weighed in air: being afterwards weighed in water, it appears to loſe the greater part of its weight.

In the early times of Philoſophy, this apparent loſs of weight was conſidered as real: and it was concluded generally, that any fluid being immerſed in a fluid of the ſame kind was deprived of its gravity or weight.

II.

Let a hollow ſphere be made heavy enough to ſink in water, and let it be weighed when immerſed: being filled with water and again weighed when immerſed, it is found to be heavier than before by the weight of the included water.

This experiment is an evident proof that a fluid when immerſed in a fluid of the ſame kind, or, as it is termed, in its own element, loſes no part of its weight. What is the occaſion of this apparent loſs of weight will appear in the articles which follow.

The fluids uſed in Hydroſtatical experiments are chiefly mercury, water, and air; theſe being the moſt free from tenacity and coheſion of ſome oils alſo are ſufficiently limpid for the purpoſe of making theſe experiments on the principles of Hydroſtatics.

The following proportions are very frequently referred to in Hydroſtatics, and are therefore here inſerted although they are contained in Art. (XXIII.) of Mechanics.

III.

1. The denſities or ſpecific gravities of bodies are in the ſame proportion with their weights, when the magnitudes of the bodies are equal.

2. When the weights of bodies are equal, the ſpecific gravities are inverſely as their magnitudes.

3. When the ſpecific gravities are equal, their weights are directly as their magnitudes.

Theſe relations may be expreſſed thus: let $\frac{W}{w}$ repreſent the ratio of the weights of two ſubſtances, $\frac{M}{m}$ the ratio of their magnitudes, and $\frac{N}{n}$ that of their denſities: the relation of theſe quantities will be expreſſed by the following equation: $\frac{W}{w} = \frac{M}{m} \times \frac{N}{n}$ in general: in eſtimating the weights magnitudes and ſpecific gravities of ſubſtances, ſome ſtandard quantities are always aſſumed to which other bodies are referred: w, m, and n are theſe ſtandards: and each of them might be aſſumed $= 1$. But as the aſſumption would not correſpond with the meaſures or weights now in uſe; it will be more eligible that *m* ſhould repreſent the magnitude of ſome known meaſure in common uſe, ſuch as a cubic inch, a cubic foot, &c. *n* may be conveniently aſſumed $= 1, 10, 100$ &c. or any number in that progreſſion: rain water is generally uſed as a ſtandard ſubſtance to which the ſpecific gravities of all other bodies are referred The denſity of rain water being found more uniformly the ſame than that of any other ſubſtance either ſolid or fluid. *w* therefore will be the weight of a cubic inch or cubic foot of rain water. It is remarkable that a cubic foot of rain water ſhould weigh exactly 1000 ounces averdupoiſe. If then we make $w = 1000$, $m = 1$, and $n = 1000$, we ſhall obtain $W = M \times N$, that is, the weight of any body in averdupoiſe ounces will be equal to the magnitude expreſſed in cubic feet multiplied into the ſpecific gravity taken from that ſcale in which the ſpecific gravity of rain water is 1000. Thus the weight of 3 cubic inches or $\frac{3}{1728}$ parts of a cubic foot of mercury will be $= \frac{3}{1728} \times 13600 = 23.61$ averdupoiſe oz.

It will be often convenient to expreſs the magnitudes in other dimenſions; *m* being always aſſumed $= 1$: let the weight of *m* be expreſſed in general by *c* which will give $W = c \times M \times N$; *n* being $= 1$.

In philoſophical ſubjects the weights of bodies are moſt frequently expreſſed either in grains, or troy ounces, the magnitudes being referred to that of a cubic inch, as a ſtandard.

If *c* be put to denote the weight of a cubic inch of rain water $= 253$-18 grains, or $.52746$ parts of a troy ounce, then $W = 253.18 \times MN$ grains or $.52746 \times MN$ troy ounces.

IV.

To determine the magnitude of an irregular ſolid.

This article is intended only to exemplify the application of the rule contained in the preceding note. If the ſpecific gravity of the irregular ſolid is s, then $s \times 253.18$ will be the weight of a cubic inch conſiſting of the ſame ſubſtance; let w be the weight of the ſolid expreſſed in grains: the following proportion will determine its magnitude.

As $s \times 253.18 : w :: 1 : x$:

wherefore $x = \frac{w}{s \times 253.18}$ being the magnitude of the ſolid eſtimated in cubic inches.

V.

To determine the capacity of an irregular veſſel.

Let the veſſel be filled with water: and let the weight of the water be A ounces: then make the following proportion: as .52746 is to A, ſo is 1 to the capacity of the veſſel expreſſed in cubic inches. The following table will facilitate the eſtimation of capacities.

oz.		cubic inches.
1	=	1.8959
2	=	3.7918
3	=	5.6877
4	=	7.5835
5	=	9.4794
6	=	11.3753
7	=	13.2712
8	=	15.1671
9	=	17.0630

To exemplify the uſe of this table, ſuppoſe the water contained in a receiver of an air pump or other veſſel to weigh 235.18 oz.

Then referring to the table.

oz.		cubic inches.
200	=	379.18
30	=	56.88
5	=	9.48
.1	=	.19
.08	=	.15
The number of cubic inches contained in the veſſel.	=	445.88

If averdupoiſe ounces are uſed in weighing the water, the numbers may be taken from the above table, but the reſulting number muſt be multiplied into (.91145) to give the true number of cubic inches contained in the veſſel.

VI.

Let mercury be poured into tubes open at both ends; ſome of the tubes being ſtrait and others bent at various angles: on immerſing the lower extremities of the tubes in water, the mercury will aſcend toward the upper ends of the tubes.

The diameters of the ſtrait tubes muſt be ſmall; and one of the extremities ſhould be cloſed till after the immerſion, to prevent the mercury from falling out. This is an eaſy and obvious method of proving that fluids preſs in all directions.

It is be remarked concerning this experiment, that whatever be the angles at which tubes are bent, and however they are inclined to the horizon, if the mercury in all the tubes before immerſion is on a level, it will continne ſo after immerſion provided the tubes are all immerſed to the ſame depth; when therefore it has been proved, that the preſſures of a fluid are as the ſurfaces preſſed, and their depths from the ſurface of the incumbent fluid, it will follow that the preſſure of a fluid is not only propagated in all directions, but that the quantities of the preſſure at the ſame depths and on a given ſurface are equal in all directions.

VII.

Let mercury be included in a tube open at both ends and let the tube be immerſed perpendicularly in water, to a ſufficient depth: the mercury will riſe in the tube; and the diſtance between the ſurface of the water in the tube, where it is contiguous to the mercury, and the ſurface of the external water, will be about 13.6 times greater than the altitude of the column of mercury in the tube.

The diameter of the tube muſt be ſmall, and the extremity ſhould be cloſed, till after the immerſion, to prevent the mercury from falling out.

This is an eaſy and obvious method of proving that fluids preſs on a given baſe with forces which are as their altitudes and ſpecific gravities jointly.

On a curſory view, it may ſeem a mechanical parodox that the preſſure of a fluid upward or in a direction contrary to that of gravity, ſhould be nothing more than a conſequeuce of gravity itſelf: but it is very eaſy to ſhew from Mechanical principles that a force acting in a given direction may communicate preſſure through a number of intermediate bodies, ſo that the laſt body ſhall be impelled in any direction whatever, even in that which is directly contrary to the original impulſe: and this is the caſe in reſpect of the particles which compoſe fluids.

VIII.

Let a piece of cork be fitted to the bottom of a veſſel ſo that the ſurfaces may be every where in contact. If mercury is poured into the veſſel, the cork will not aſcend till it is ſeparated from the bottom of the veſſel.

The effects of a fluid's preſſure in a direction contrary to that in which gravity acts are here evinced by a very deciſive experiment; as long as the fluid is prevented from communicating with the under ſurface, the cork continues attached to the bottom of the veſſel partly by its own weight and partly by the preſſure of the mercury on its upper ſurface.

IX.

The quantity of preſſure ſuſtained by any ſurface immerſed in a fluid, is equal to the weight of a column of a fluid the baſe of which is the ſurface preſſed, and the altitude equal to the diſtance between the ſurface of the fluid and the centre of gravity of the ſurface preſſed.

In this propoſition the preſſure on each point of the ſurface is eſtimated in a direction perpendicular to it. But it is often required to find the preſſure in a vertical direction: if the ſurface is a concave veſſel containing a fluid, the vertical preſſure will be equal to the weight of the fluid. When a ſolid is immerſed in a fluid, ſuppoſe that any horizontal ſection divides the ſolid into two parts; this ſection will be preſſed by the fluid downward with a force equal to the weight of a column of the fluid, of which the baſe is the ſection itſelf, and the altitude its diſtance from the ſurface of the fluid, this weight being diminiſhed by the weight of a quantity of the fluid, equal in bulk to the ſolid above the ſection: the preſſure upward againſt the ſame ſection, will be equal to the weight of the column before deſcribed, added to the weight of a quantity of the fluid equal in bulk to the ſolid below the ſection.

Particular caſes of this propoſition admit of very exact illuſtration from experiment.

X.

If a hollow cube is filled with any fluid, the preſſure ſuſtained by each ſide is equal to half the weight of the fluid.

There is a particular point in which the whole preſſure againſt the ſide acts: it is called the centre of preſſure, and is the ſame with the centre of oſcillation of the ſide vibrating on the upper line of it as an axis. The diſtance of this point from the top is two thirds of the cubes height.

XI.

If any number of tubes or other cavities however differing in dimenſion and figure communicate with each other, ſo that a fluid being poured into one of them may flow into the others, the ſurfaces of the fluid in them all are on a level.

XII.

If two immiſceable fluids are included in a bent tube of ſuch dimenſions that the fluids may remain in ſeparate columns, their perpendicular altitudes eſtimated from an horizontal plane drawn through their common ſurface where they are in contact, will be inverſely as their ſpecific gravities.

XIII.

Let a circular plate of braſs be exactly fitted ſo as to cover the lower aperture of a cylindrical tube immerſed perpendicularly in water. If the plate be immerſed to a depth of eight times its own thickneſs, it will be juſt ſupported by the preſſure of the water.

The braſs endeavours to deſcend by its own weight. It is preſſed upward by a weight equal to that of a cylindrical column of water having the ſame baſe with the braſs plate, and being eight times thicker: and becauſe braſs is eight times ſpecifically heavier than water, the weight of the cylindrical column of water which preſſes upward and the weight of the braſs by which it endeavours to deſcend will be equal; and conſequently the braſs will be juſt ſupported in equilibrio.

XIV.

Let a tube open at both ends be inſerted in a cloſed cylindrical veſſel: if water is poured in ſo as to riſe in the tube to any altitude, the preſſure which the baſe of the veſſel ſuſtains is equal to the weight of a cylindrical column of water, the baſe of which is the ſurface preſſed upon, and the altitude equal to that of the water in the tube.

It appears that the preſſure on the baſe may be greater than the weight of water which cauſes the preſſure in any proportion: this confirms the general propoſition that the preſſure from a fluid depends on the ſurface preſſed and the altitude of the fluid above its centre of gravity only; not being at all altered by the figure or dimenſions of the veſſel, nor by the quantity of the fluid contained in it, provided the baſe which is preſſed, and the diſtance of its centre of gravity from the ſurface of the fluid remain the ſame.

XV.

If a ſolid floats on the ſurface of a fluid, the weight of the fluid diſplaced is equal to that of the ſolid.

It is here ſuppoſed that the air is removed from the ſurface of the fluid.

The following concluſions are derived from this principle. Becauſe the weights of two bodies being equal, their ſpecific gravities are inverſely as their magnitudes, it will follow in reſpect of the floating ſolid, that as the part immerſed is to the whole, ſo is the ſpecific gravity of the ſolid to that of the fluid.

A ſhip diſplaces a quantity of water equal in weight to the ſhip and its loading: if therefore the quantity of water diſplaced can be any how aſcertained, the weight of the ſhips and its contents will be known.

XVI.

Let a cylinder of elm (or other ſubſtance of the ſame ſpecific gravity), ten inches in length, be immerſed perpendicularly in water: four inches of the cylinder will remain above the ſurface. If the ſame cylinder is immerſed in ſpirits of wine, only three inches will remain above the ſurface.

The hydrometer is conſtructed on this principle. It is an inſtrument uſed to aſcertain the ſpecific gravities of fluids. The weight of the hydrometer continuing the ſame, and equal to the weights of the fluids diſplaced by it, the ſpecific gravities of thoſe fluids will be inverſely as the parts of the hydrometer which are immerſed in them: thus the depth to which it ſinks in water was obſerved to be 6 inches, in ſpirits of wine ſeven inches; wherefore the ſpecific gravity of water is to that of ſpirits of wine as 7 to 6 or as 1 to .857.

XVII.

If a veſſel contains two immiſceable fluids (ſuch as water and mercury) and a ſolid of ſome intermediate ſpecific gravity is immerſed under the ſurface of the lighter fluid and floats on the heavier; the part of the ſolid immerſed in the heavier fluid, is to the whole ſolid as the difference between the ſpecific gravities of the ſolid and lighter fluids to the difference between the ſpecific gravities of the two fluids.

Let the ſpecific gravity of the heavier fluid be a, the part of the ſolid immerſed in it $= A$: let the ſpecific gravity of the lighter fluid be $= b$, the part of the ſolid immerſed in it $= B$, and let the ſpecific gravity of the ſolid be c: alſo let the area of the horizontal ſection of the ſolid where the two fluids are contiguous be S, and the perpendicular diſtance of it from the ſurface of the lighter fluid be $= p$. Then the preſſure againſt the ſection from the lighter fluid will be $Sp - Bb$ which added to the weight of the ſolid $\overline{A+B} \times c$, will give the whole force by which this ſection of the ſolid endeavours to deſcend $= Sp - Bb + \overline{A+B} \times c$: the preſſure upward againſt the ſame ſection is $Sp + Aa$: but as the ſolid is ſuſtained in equilibrio, theſe two contrary forces muſt be equal: wherefore $Sp - Bb + \overline{A+B} \times c = Sp + Aa$; and $Ac + Bc = Aa + Bb$ or $Aa - Ac = Bc - Bb$ and conſequently $A : B :: c - b : a - c$; or as $A : A+B :: c - b :: a - b$.

XVIII.

Let a cylinder ſeven inches in length, the ſpecific gravity of which is to that of mercury as 9 to 14, be immerſed in mercury: 2.5 inches of it will remain above the ſurface. If water is poured on the mercury, till it covers the cylinder, 2.7 inches will remain above the ſurface.

In the firſt part of this experiment, what part of the cylinder is immerſed, appears from the proportion in Art. XVI. as the part immerſed, is to 7 ſo is 9 : 14, wherefore the part immerſed $= 4.5$ inches, and conſequently that which remains above the ſurface $= 2.5$ inches. The effect of the water in elevating the cylinder, is deduced from the laſt article, which being applied to this experiment, will give $A+B = 7$ (the parts of the cylinder being as their lengths), $c=9$, $a=14$, $b=1$, and $A : A+B\ (7) :: c-b\ (8) : a-b$

$= 13$; wherefore $A = \frac{56}{13} = 4.3$, and conſequently B

$= 2.7$.

The ſpecific gravity of mercury is here aſſumed $= 14$ inſtead of 13.6, that the example might be more obvious: the difference will not be of any ſenſible effect.

From what has preceded it is evident that if any ſubſtance floats on the ſurface of a fluid in vacuo, on admitting the air, the ſubſtance will riſe higher above the ſurface, and conſequently the proportion of the part immerſed to the whole will be ſomething leſs than before: but the difference is very inconſiderable, being only that part of the ſolid which is expreſſed by the fraction

$\frac{b \times \overline{a-c}}{a^2}$; where b is the air's ſpecific gravity, a the ſpecific gravity of the fluid, and c that of the ſolid which floats on it. For which reaſon, as far as regards the proportion of the part of any ſolid immerſed in a fluid on which it floats to the whole, the air's preſſure need not be taken into account.

XIX.

A ſolid weighs leſs when immerſed in a fluid than when it is weighed in open air, the weight loſt being that of a quantity of the fluid equal in bulk to the ſolid.

In ſtrictneſs the ſolid ſhould be weighed in vacuo, to obtain its true weight; the fluid in which it is afterwards immerſed and weighed ſhould alſo be in vacuo. The error cauſed by the air's preſſure in eſtimating ſpecific gravities is very ſmall, and need not be allowed for, except in particular caſes when great preciſion is required.

From the principle expreſſed in the article, the ſpecific gravities of ſolids in reſpect of a given lighter fluid are immediately determined: For, having the loſs of weight by immerſion in the fluid, the weight of an equal bulk of the fluid is known; and when the magnitudes of two bodies are equal, their ſpecific gravities are as their weights, (Art. III.) which may be expreſſed by this analogy; as the ſolid's weight in air is to the difference of the weights in air and in the fluid, ſo is the ſpecific gravity of the ſolid to that of the fluid. Thus ſuppoſe a piece of braſs weighed in air 88 grains, in water only 77 grains; then the difference of the weights in air and water would be 11 grains, and the proportion would be; as 88 : 11 ſo is the ſpecific gravity of braſs to that of water, or as 8 : 1.

If the ſubſtance of which the ſpecific gravity is required conſiſts of ſmall fragments or of minute particles, it muſt be placed in a veſſel and then weighed in a proper fluid, allowance being made for the weight loſt by the veſſel; the weight loſt by the ſubſtance itſelf will be known, and conſequently its ſpecific gravity determined.

If powder conſiſts of round and equal particles, the ſpecific gravity of the ſolid particles might be had from weighing a given quantity of it, for inſtance a cubic inch: it is a property of very minute ſpherical bodies when they are placed the cloſeſt poſſible in ſolid ſpace, that the interſtices, or empty ſpace, ſhall be to the whole bulk as $\sqrt{18}-p$ to $\sqrt{18}$, p being = 3.14159. Conſequently if the weight of a cubic inch of thoſe ſpheres = w, the weight of the ſame magnitude without empty ſpace would be $w \times \frac{\sqrt{18}}{p}$ and the ſpecific gravity of the ſolid ſpherical particles would be thereby determined.

The ſpecific gravities of fluids are uſually conſidered without any regard to the empty ſpaces between the particles, though, however minute they may be, it is probable that the empty ſpace is more than $\frac{1}{4}$ part of the whole bulk. But it is ſufficient that we know preciſely in what ſenſe the ſpecific gravity of fluids is underſtood.

The ſpecific gravity of fluids is determined by the hydrometer, or more accurately in many caſes by the method contained in the next article.

XX.

A cubic inch of any ſubſtance being immerſed in water weighs leſs by 253.18 grains, than when it is weighed in air. If the cubic inch is immerſed in ſpirits of wine, it weighs leſs by 217 grains than when it is weighed in air.

It appears that the weight of a cubic inch of water is 253.18 grains, and that a cubic inch of ſpirits of wine weighs 217 grains; wherefore the ſpecific gravities of water and ſpirits of wine are as 253.18 : 217 or as 1 to .857; as was before found. (Art. XVI.)

There is another methods by which the ſpecific gravities of fluids are determined: which is, by filling a veſſel with the two fluids ſucceſſively, and finding the weights of the contents: the ratio of the weights will be that of the ſpecific gravities.

XXI.

The ſpecific gravity of a body which will not ſink in water or other given fluid, may be determined by uniting it to a ſolid ſo that the compound may ſink in the fluid.

The weight loſt by immerſing the heavier ſolid in water may be found by weighing it ſeparately in air and water: let this weight loſt $= A$; the weight loſt by the compound may be found by weighing it in air and water, let this $= B$: then $B - A$ will be the weight of a quantity of water equal in bulk to the lighter ſolid, of which the ſpecific gravity is required, which will be to that of water as the weight of the ſolid is to the difference of the weights $B - A$.

Or the ſpecific gravity of the ſolid may be determined thus:

Let A be the weight of the heavier ſolid.
a its ſpecific gravity.
B the weight of the lighter ſolid.
x its ſpecific gravity ſought.
C the weight of the compound.
c its ſpecific gravity.

From the principles of ſpecific gravities it is known that $\frac{A}{a} + \frac{B}{x} = \frac{C}{c}$, and by reducing this, $x = \frac{Bac}{Ca - Ac}$.

XXII.

When any ſolid is immerſed in a fluid, the ſame weight which loſt by the ſolid, is added to the fluid in which it is immerſed.

This follows from the Statical axiom, that action and reaction are equal and in contrary directions.

If a veſſel containing water is counterpoiſed in a balance, and a cubic inch of any ſubſtance is immerſed in it, the cube will loſe the weight of a cubic inch of water, which has appeared to be 253.18 grains; and the veſſel of water will gain juſt 253.18 grains, the veſſel which contains the water preponderating; the equilibrium is reſtored by adding 253.18 grains to the oppoſite arm of the balance.

XXIII.

If a piece of copper is exactly counterpoiſed in air by a braſs weight, and both the copper and braſs are immerſed in water; the copper will preponderate.

Let w be the weight of either ſolid, c the ſpecific gravity of the copper, b that of the braſs, a that of the fluid in which they are immerſed, then the braſs will loſe by immerſion a weight $= \frac{wa}{b}$ and the copper willl loſe $\frac{wa}{c}$, and the difference of theſe will be a weight p by which the equilibrium is deſtroyed; wherefore $p = \frac{wa}{b} - \frac{wa}{c} = wa \times \frac{\overline{c-b}}{cb}$; reducing this to an analogy, it will be, as the product of the ſpecific gravities, is to their difference multiplied into the ſpecific gravity of the fluid in which they are immerſed, ſo is the weight of either ſolid, to the weight which muſt be added (not immerſed) to reſtore the equilibrium. Thus if 120 grains of braſs are counterpoiſed in air by a piece of copper, and both are immerſed in water, by referring to the rule it will be known what weight muſt be applied to the aſcending arm of the balance to reſtore the equilibrium.

$$\text{As } 9 \times 8 : \overline{9-8} :: 120 : \frac{120}{72} = 1 \text{ grain and } \tfrac{2}{3}.$$

If a guinea weighing 129 grains be counterpoiſed by a piece of braſs, and both the gold and braſs are immerſed in water, $\frac{129 \times \overline{17.2-8}}{8 \times 17.2}$ or ſomething more than 8½ grains muſt be added to the arm of the balance from which the braſs is ſuſpended to reſtore the equilibrium. Here the ſpecific gravity of ſtandard gold is aſſumed at 17.2 when that of water is 1.

ON THE WEIGHT AND ELASTICITY OF THE AIR.

XXIV.

If a veſſel be weighed when full of air and when exhauſted, the difference of the weights will be about 324 grains for a thouſand cubic inches.

Conſequently 1728 cubic inches or a cubic foot of air will weigh 559.87 grains, or 1.166 oz. troy. Becauſe a cubic inch of water weighs 253.18 grains and a cubic inch of air weighs .324 parts of a grain, the ſpecific gravity of water will be to that of air as 781.4 to 1, and the ſpecific gravity of mercury to that of air as $781.4 \times 13.6 = 10627$ to 1.

This is the air's denſity correſponding to the temperature of the freezing point, when the barometer is at 30 inches.

As the air poſſeſſes weight in common with all other fluids, it muſt preſs on the ſurfaces of bodies immerſed in it: the quantity of this preſſure is next to be conſidered.

XXV.

Let a glaſs tube or veſſel of any ſhape thirty ſix inches in length, one extremity of which is cloſed, be filled with mercury: let the other extremity be immerſed in a veſſel of mercury: the mercury in the tube or veſſel will deſcend from the upper extremity, and will remain ſuſpended at ſome altitude, between 28 and 31 inches from the ſurface of the external mercury.

This experiment is uſually made with a tube of a cylindrical ſhape, which, being filled and immerſed in the manner deſcribed in the article, is called a barometer.

The principle on which the ſuſpenſion of the mercury depends, is extended to veſſels or tubes of every ſhape and dimenſion, and inclined at any angle to the horizon.

To deſcribe this principle fully, particular attention ſhould be paid to the horizontal ſection of the tube which is on a level with the ſurface of the external mercury. All the mercury beneath this ſection both within and without the tube is counterpoiſed, and can therefore contribute nothing to ſupport the mercury above it, nor can at all add to its weight. The preſſure on this ſection from the mercury ſuſtained, will be equal to the weight of a column of mercury, the baſe of which is the ſection, and altitude equal to that of the mercury above the ſection: this will be true whatever be the ſhape and dimenſions of the ſection and of the veſſel or tube containing the mercury.

From the ſame principle, the ſection before referred to, will be preſſed upward by the weight of a column of air, the baſe of which is the ſection, and altitude that of the air, ſuppoſing the whole atmoſphere were of the ſame uniform denſity; and becauſe the mercury is ſuſtained in equilibrio, the preſſures or weights of the two columns are equal, and conſequently their altitudes inverſely proportional to their ſpecific gravities.

From this determination the height of an homogeneal atmoſphere is inferred, the ſpecific gravities of air and mercury being given; and the altitude of the mercury in the barometer correſponding; the proportion will be this: as the ſpecific gravity of air (1) is to the ſpecific gravity of mercury 10627, Art. XXIV. ſo is the altitude of the mercury in the barometer, 30 inches or 2½ feet, to the height of an homogeneal atmoſphere, which will be found = 26567 feet.

The altitude of the mercury is a very accurate meaſure of the air's weight or preſſure, which, as is well known, varies conſiderably; the greateſt difference being near one part in ten of the whole. The air's preſſure on any given ſurface when the altitude of the barometer is 30 inches, will be at the rate of 30 cubic inches of mercury or about 15 pounds averdupoiſe, for every ſquare inch of ſurface preſſed.

It muſt be remarked, that the height of an homogeneal atmoſphere is not at all altered by any difference in the air's weight: let b be the height of an homogeneal atmoſphere; a the altitude of the mercury in the barometer; the ſpecific gravities of mercury and air; as $n : m$ then by what has preceded, as $n : m :: b : a$, and $b = \frac{na}{m}$; but when the ſpecific gravity of the air m alters, that of mercury $= n$ continuing the ſame, m is always proportional to a, that is the air's preſſure is always proportional to the height of the mercury in the barometer; wherefore $\frac{na}{m}$ or b muſt be always a given quantity; the effects of heat and cold not being here conſidered.

The altitude, at which the mercury is ſuſtained in the barometer above the ſurface of the external mercury, is called the ſtandard altitude.

XXVI.

Let a tube or veſſel of any ſhape or dimenſion be filled with mercury and immerſed when inverted in a veſſel of mercury: if the altitude of the extremity of the tube or veſſel be leſs than the ſtandard altitude, the mercury will remain contiguous to the extremity.

In this caſe the air's preſſure is greater than the weight of the column of mercury ſuſtained by it.

XXVII.

If tubes of various dimenſions and figures be filled with mercury and immerſed when inverted in the ſame veſſel of mercury at any different inclinations and poſitions in reſpect of the horizon, the upper ſurfaces of the mercury ſuſtained in them all will be on a level.

The diſtance between the horizontal planes which coincide with the upper and lower ſurfaces of the mercury reſpectively will be equal to the ſtandard altitude.

The increaſe of ſcale gained in the diagonal barometer ſeemed to make that conſtruction preferable to the vertical or common barometer; but the want of exactneſs in the termination of the mercury's ſurface, by which its height is denoted, has been found more than ſufficient to counterbalance the advantage of an enlarged ſcale. The diagonal barometer is now almoſt wholly diſuſed. According to the modern method of conſtructing barometers, the height of the mercury can be aſcertained true to $\frac{1}{500}$ of an inch, or about the $\frac{1}{15000}$ part of the whole when the barometer is at 30 inches.

XXVIII.

Let a barometer be placed under the receiver of an air-pump: as the air is exhauſting, the mercury continually deſcends till it is nearly on a level with the mercury in the veſſel. Upon admitting the air, the mercury riſes to its former altitude.

The air's preſſure being the power by which the mercury is ſuſtained by the barometer, it is a conſequence that on the removal of the air from the mercury's ſurface, the mercury in the tube muſt deſcend. So far this experiment confirms what has been advanced concerning the effects of the air's preſſure.

A barometer included in the receiver of an air-pump is of great uſe in aſcertaining the preſſure of the air remaining in it after having been partly exhauſted. For this preſſure will always be to the preſſure of the atmoſphere on a given ſurface, as the height of the mercury in the tube is to the ſtandard altitude. The barometer applied to this uſe is ſometimes called a gage.

XXIX.

If a barometer, the tube being a veſſel of any ſhape, be immerſed in a veſſel

of

of mercury, and ſuſpended from the beam of a balance, the weight neceſſary to counterpoiſe it, excluſive of the tube, is equal to the weight of the mercury contained in the barometer.

To explain this, ſuppoſe the tube to be a cylinder or priſm. It may ſeem that the arm of the balance ought to ſuſtain no more weight than that of the tube, ſince the mercury is ſuſtained by the preſſure of the atmoſphere; but it is to be conſidered that, when the tube is empty, the air preſſes equally on every part of its ſurface both the inſide and outſide of it: but when it is filled with mercury and ſuſpended in the manner deſcribed in the article, the air ſtill preſſes on the outſide; and it preſſes directly upward againſt the lower edge of the tube, this preſſure being communicated through the mercury: but the preſſure againſt the horizontal ſection in the tube deſcribed in the note to Art. XXV. is totally counteracted by the weight of the mercury ſuſtained by it: and it follows, that the air does not preſs at all upward on a ſpace at the top of the tube eſtimated horizontally equal to the horizontal ſection underneath, conſequently the atmoſpheres preſſure on the outſide of the tube will act on it with the ſame force, as on the horizontal ſection below, that is with a force equal to the weight of the mercury ſuſtained in the tube.

XXX.

Let a tube thirty ſix inches in length, open at both ends, be immerſed in mercury, contained in a veſſel, from the inſide of which, the external air is excluded:

if the tube and veſſel be placed under a receiver, on exhauſting the air, the mercury is raiſed in the tube nearly to the ſtandard altitude.

The air, beſides the gravity or weight which it poſſeſſes in common with all fluids, is endowed with a remarkable property by which a given quantity of it may be compreſſed into ſmaller dimenſions, or may occupy a larger ſpace than that which it poſſeſſes in the atmoſphere; obſerving very exact laws during its compreſſion or expanſion which will appear in the ſubſequent articles.

The experiment juſt deſcribed is intended to ſhew that the air in its natural ſtate in the atmoſphere exerts a force of elaſticity equal to the force which compreſſes it; this indeed appears evident from the firſt principle of Statics, that action and reaction are equal; for admitting the air to be in a ſtate of equilibrium with itſelf, there muſt neceſſarily exiſt a force in any given quantity of air, which reſiſts and exactly counterpoiſes the external preſſure.

It is from this principle that in any cloſed unexhauſted veſſel or receiver which removes the incumbent preſſure of the external air from ſubſtances placed within it, the ſubſtances ſo placed ſuſtain a preſſure from the air's elaſtic force, equal to the weight of the atmoſphere preſſing againſt the ſame ſurface.

XXXI

The ſpaces into which a given quantity of air is compreſs'd, are leſs in the ſame proportion as the forces of compreſſion are greater.

That experiments on the air's elaſtic force in reſpect of its ondenſation may be ſatisfactory, a receiver of an air-pump ſhould be uſed, the capacity of which is exactly known: let a veſſel of mercury be placed in this receiver, and let a tube of which both ends are open be immerſed in the mercury, being continued through the receiver; the magnitude of the veſſel and the mercury contained in it, and the portion of the included tube, muſt be found and ſubtracted from the capacity of the receiver: Let the remaining capacity be denoted by A: then a known quantity of atmoſpheric air B being thrown into the receiver, the mercury will riſe, according to the following proportion: as the elevation of the mercury is to the ſtandard altitude, ſo is the air thrown into the receiver, to that which it poſſeſſed before any air was added to it; or if S be the ſtandard altitude and H the height of the mercury in the tube: as H : S : : B : A.

From hence it is inferred, that the quantity of air in the receiver will be always proportional to the ſtandard altitude, added to the altitude of the mercury in the tube, which two columns meaſure the quantity of the air's preſſure on the ſurface of the mercury in the receiver, or in other terms the preſſure of the air on a given ſurface is as its denſity.

This method of aſcertaining the air's elaſtic force is very exact, but is ſuited to a moderate condenſation of the air only. If it is propoſed to obſerve the elaſtic force of air compreſſed in a very great degree, other methods are neceſſary, which however are not ſo precise as that which has been deſcribed.

XXXII.

A given quantity of air being expanded into larger dimenſions exerts on the ſame ſurface an elaſtic force, which is leſs in the ſame proportion as the ſpace occupied by the air is greater.

Suppoſe a receiver to contain a mercurial barometer, and the capacity of it to be exactly known, due allowance being made for the magnitude of the barometer: ſuppoſe this capacity to be A; let the receiver, when full of the common atmoſpheric air, communicate with an exhauſted receiver of which the capacity is B; then the mercury in the barometer will gradually ſubſide until it remains ſuſpended at an altitude which is to the ſtandard altitude as A to A + B, that is the altitudes of the mercury will be inverſely proportional to the ſpaces occupied by the given quantity of air contained originally in the receiver A.

Some allowance muſt be made for the ſmall remains of air in the exhauſted receiver.

The barometer gage of an air-pump will denote not only the quantity of the air's preſſure Art. XXVIII. but that of its denſity and elaſtic force alſo.

XXXIII.

If any portion of a barometer tube exactly cylindrical be occupied by air before inverſion, after the barometer is inverted, the mercury will ſubſide below the ſtandard altitude; and the ſtandard altitude will be to the defect from it, as the ſpace occupied by the air after inverſion, to the ſpace occupied by it before inverſion.

This follows from the decreaſe of the air's elaſtic force in proportion to the ſpace occupied by it. Before the air is expanded, its elaſticity is equal to the weight of a column of mercury the height of which is the ſtandard altitude preſſing againſt the ſame ſurface. After inverſion the elaſticity muſt be equal to the weight of a column of mercury which is the defect from the ſtandard altitude, and the ſpaces occupied by the air being inverſely ly as their elaſtic forces, it will follow that the ſtandard altitude will be to the defect from it, as the ſpace occupied by the air after inverſion to the ſpace occupied by it before inverſion.

Let s repreſent the ſtandard altitude, $d =$ the defect from it, $b =$ the difference between the ſtandard altitude, and that part of the tube which is above the ſurface of the external mercury, $q =$ the quantity of air included. Then the following proportion will be general.

As $s : d :: b + d : q$, and $q = \frac{\overline{b+d} \times d}{s}$ by which the ſtandard altitude and the defect from it being given, with the length of the tube above the external mercury, the quantity of air included will be known.

If the quantity of included air be given to find the defect from the ſtandard altitude, let $x =$ the defect ſought, and by the preceding rule $s : x :: b + x : q$, wherefore $x^2 + bx = sq$ and $x = \frac{\sqrt{b^2 + 4sq} - b}{2}$.

If the ſtandard altitude is greater than the length of the tube, the ſign of b will be changed, and x or the defect from the ſtandard altitude will now $=$ $\frac{\sqrt{b^2 + 4sq} + b}{2}$.

The barometer tube being cylindrical, the quantity of air included before inverſion and the ſpaces occupied by it after inverſion will be truly eſtimated by the degrees into which the length of the tube is divided.

The general concluſion deduced from the two latter articles is, that the elaſtic force of the air is always proportioned to its denſity and to its preſſure on a given ſubſtance.

XXXIV.

The altitude of the mercury in a barometer elevated above the earth's ſurface, is obſerved to be leſs than the ſtandard altitude at the earth's ſurface.

Since the ſuſpenſion of the mercury in the barometer is cauſed by the preſſure or weight of the atmoſphere, when any part of that weight is taken off, the altitude of the mercury will be leſs: if a barometer is elevated above the earth's ſurface; all the column of air beneath the barometer, which before the elevation contributed to keep the mercury ſuſtained, will now no longer have any effect on it; conſequently the weight of the atmoſphere, meaſured by the altitude of the barometer at the earth's ſurface, will be diminiſhed at the upper ſtation by the weight of the intermediate column, and the altitude of the mercury in the barometer will be proportionally diminiſhed, the diminution being a column of mercury, of which the weight is equal to that of the intermediate column of air between the earth's ſurface and the elevated barometer.

It is evident, ſince the preſſure of the air on a given ſurface is continually leſſened as we aſcend in the atmoſphere, that the denſity muſt alſo decreaſe in the ſame proportion: Art. XXXII. the altitude of the mercury in the barometer will therefore be a meaſure of the air's preſſure and elaſtic force at different heights above the earth's ſurface at any given time, as well of the variations of the air's weight and elaſtic force at a given height, obſerved at different times.

XXXV.

The height of an atmoſphere which is uniformly of the ſame denſity with the air at any altitude above the earth's ſurface is a given quantity, being equal to the height of an homogeneal atmoſphere of the ſame denſity, with the air at the earth's ſurface.

Let the ſpecific gravity of air be to that of mercury, as $n : 1$. Let h be the height of an homogeneal atmoſphere at any given ſtation: let $a =$ the altitude of the mercury in the barometer at the ſame ſtation: then by what has preceded $n : 1 :: a : h$, and $h = \frac{a}{n}$: but a will be always a meaſure of the air's denſity and preſſure, and will therefore always vary in the ſame proportion with a; conſequently $\frac{a}{n}$ or h will be a given or conſtant quantity, at all altitudes from the earth's ſurface.

XXXVI.

Two altitudes of the barometer *a* and *b* correſponding to the two ſtations A and B being given, to find the difference of the altitudes of A and B from the earth's ſurface.

Let h be the height of an homogeneal atmoſphere; let the vertical diſtance between A and B be x, A being the lower ſtation: let d be the denſity at B, when the denſity at A is t. Suppoſe the altitude x to increaſe by the leaſt increment $\dot{x}$, then the decrement of preſſure correſponding to the ſpace $\dot{x}$, will be to the whole preſſure at B as $\dot{x} : b$; the weights of columns that are uniform and ſtand on the ſame baſe being as their altitudes: but as the decrement of preſſure is to the whole preſſure at B, ſo is the decrement of the denſity to the denſity at B; wherefore as $\dot{x} : b :: -\dot{d} : d$, and $\dot{x} = -b \times \frac{\dot{d}}{d}$ or $x = -b \times \log.\ d = -b \times \log. \frac{b}{a}$ or $x = b \times \log. \frac{a}{b}$ expreſſed in hyperbolic logarithms: if the tabular logarithms are uſed, x will $= 2.3058 \times b \log. \frac{a}{b}$ or $x = 61172 \times \log. \frac{a}{b}$ in feet.

If the air were uniformly at the temperature of the freezing point at all altitudes from the earth's ſurface, this rule would require no correction, but it is known that the air's temperature is moſt commonly very different in the higher regions from that which is experienced on the earth's ſurface: this alters the proportion of the air's denſity in reſpect of its weight, and makes it neceſſary to apply a correction on account of the difference between the temperatures as indicated by the thermometer at the upper and lower ſtations.

CONCERNING PUMPS.

XXXVII.

On the operation of pumps which raiſe water by the preſſure of the atmoſphere.

A pump has no effect in raiſing water which is placed in an exhauſted receiver.

The elevation of water by the pump being cauſed by the atmoſphere's preſſure, which is equivalent to that of about 30 inches of mercury or 34 feet of water on a given ſurface, it is plain that water cannot be raiſed by a pump to an altitude greater than that of about 34 feet, this height being a little greater or leſs on account of the variation of the atmoſphere's weight.

XXXVIII.

On the conſtruction and operation of pumps which raiſe water by the force of condenſed air.

The effect of this kind of pump is not limited to raſing water to any particular altitude, ſince the air's condenſation may be increaſeto any degree.

If the air's condenſation is double to that of atmoſpheric air, its elaſtic force will raiſe water to the height of about 34 feet; if the condenſation is increaſed threefold, the altitude to which water may be raiſed by it will be about twice the former height or 68 feet: the altitude of the raiſed water being increaſed 34 feet for each addition of unity to the number which expreſſes the air's condenſation.

XXXIX.

On the operation of the pump by the rareſaction and condenſation of ſteam, joined with the atmoſphere's preſſure.

XL.

On the conſtruction and uſe of the air-pump.

XLI.

Let the capacity of the receiver be A, that of the barrel be = B: then whatever be the rareneſs of the air contained in the receiver, one elevation of the piſton will take from it a quantity which is to the whole as B is to A + B.

Let A + B be to B as $n : 1$, and let a be the quantity of atmoſpheric air in the receiver before exhauſtion: then the piſton being ſucceſſively elevated; the quantities of air exhauſted and the remaining portions will be expreſſed as underneath.

N° elevations.	Quant. exhauſted.	Quant. remaining.
1	$\frac{a}{n}$	$a\times\frac{\times\overline{n-1}}{n}$
2	$\frac{a}{n}\times\frac{n-1}{n}$	$a\times\frac{\overline{n-1}^{2}}{n^{2}}$

Nº elevations.	Quant. exhauſted.	Quant. remaining.
3	$\frac{a}{n}\times\frac{\overline{n-1}^{2}}{n^{2}}$	$a\times\frac{\overline{n-1}^{3}}{n^{3}}$
4	$\frac{a}{n}\times\frac{\overline{n-1}^{3}}{n^{3}}$	$a\times\frac{\overline{n-1}^{4}}{n^{4}}$
r	$\frac{a}{n}\times\frac{\overline{n-1}^{r-1}}{n^{r-1}}$	$a\times\frac{\overline{n-1}^{r}}{n^{r}}$.

It appears that the quantities exhauſted will be continually leſſened in the given proportion of $n : n-1$; and that the quantities remaining in the receiver will be diminiſhed in the ſame proportion. Conſequently both the quantities exhauſted and thoſe remaining in the receiver will be diminiſhed in geometrical progreſſion. Let q be the quantity remaining in the receiver after r elevations of the piſton.

Since $q = a\times\frac{\overline{n-1}^{r}}{n^{r}}$ it follows that $\frac{q}{a}=\frac{\overline{n-1}^{r}}{n^{r}}$, and

$$r=\frac{\text{Log.}\,\frac{q}{a}}{\text{Log.}\,\frac{n-1}{n}}.$$

XLII.

To determine the capacity of the tubes which form the communication between the barrel and receiver, and to determine the capacity of the barrel.

In the laſt article the tubes which form the communication between the barrel and receiver were not conſidered; in ſome experiments it is extremely uſeful to know preciſely the capacity of thoſe tubes; which is aſcertained by the enſuing proceſs, and the capacity of the barrel is at the ſame time obtained.

Let the apertures of the air-pump be cloſed, no receiver being applied; let the piſton be elevated once; and mark the altitude of the mercury in the gage, which ſuppoſe $= a$; the ſtandard altitude being $= b$. Let the capacity of the barrel be x, and that of the tubes of communication $= y$; then as $y : x+y :: a : b$, and as $x : y$ $b-a : a$; let $b-a$ be to a as m to 1.

A receiver of which the capacity is c (which ſhould not greatly exceed that of the barrel) being applied, let the piſton be elevated once; and let the altitude of the mercury in the gage be a; the ſtandard altitude being $= b$: then ſince the elaſtic force of any given quantity of air is inverſely as the ſpace it occupies, we ſhall have $\frac{a}{b} = \frac{c+y}{c+my+y}$ and $y = \frac{c\times\overline{b-a}}{a\times\overline{m+1}-b}$, wherefore $x = \frac{mc\times\overline{b-a}}{a\times\overline{m+1}-b}$ from which equations the capacity of the barrel and that of the tubes of communication are known.

SOME EXPERIMENTS ON THE EFFECTS OF THE AIR'S PRESSURE AND ELASTIC FORCE.

XLIII.

Let the ſhorter arm of a ſyphon be immerſed in mercury: if the air is removed from the inſide of the tube, it is obſerved that the mercury will flow out of the longer arm in a continued ſtream.

In this experiment the perpendicular height of the ſhorter arm above the ſurface of the mercury in which it is immerſed muſt be leſs than the ſtandard altitude of the mercury in the barometer.

If a mercurial ſyphon is included in the receiver of an air-pump the air being partly exhauſted, the mercury will not flow out of the longer arm unleſs the perpendicular height of the ſhorter arm is leſs than the altitude of the mercury in the gage which meaſures the elaſtic force of the air in the receiver.

XLIV.

Let a ſmall veſſel be filled with water, and let the aperture of it be exactly covered with a braſs or other plate: if the veſſel be inverted, the plate will ſtill continue to adhere to it.

In this caſe the preſſure of the atmoſphere in a direction contrary to that of gravity exceeds the weight of the plate and of the water which reſts on it.

XLV.

Let a thin glaſs veſſel, the aperture of which is cloſed, be placed under the receiver of an air pump: if the air is exhauſted from the receiver, the veſſel will not be broken.

XLVI.

Let a thin glaſs veſſel, the aperture of which is cloſed, be fitted with a valve opening outward: let this be placed under the receiver of an air-pump, and let the air be exhauſted: on admitting the air, the veſſel is broken.

XLVII.

Let any thin ſubſtance impervious to air be fitted to the upper aperture of a receiver ſo that there may be no communication between the internal and external air: on exhauſting the receiver, the ſubſtance is broken.

XLVIII.

Let two hollow hemiſpheres be cloſely placed together, ſo that all communication between the internal and external air may be prevented; if the air be exhauſted from the cavity, it is obſerved that the hemiſpheres are ſo preſſed together, that a force equivalent to about fifteen pounds for every ſquare inch of a great circle of the hemiſpheres is required to ſeparate them.

XLIX.

If an exhauſted receiver be made to communicate with a veſſel of water by means of a tube inſerted in the centre of the plate on which the receiver ſtands, the water aſcends through the tube, and ſtrikes againſt the extremity of the receiver with a continued ſtream.

SOME PROPERTIES OF ATMOSPHERIC AIR.

L.

If a bell be ſtruck under the receiver of an air-pump, it is obſerved that the ſound becomes weaker the more perfectly the air is exhauſted.

It appears from this experiment that ſound is propagated by the air: ſound is alſo conveyed by continued media of ſeveral kinds: if two hard ſubſtances are ſtruck together under water, the ſound will be heard by a perſon immerſed under water at a greater diſtance than the ſound can be heard in the open air. If the ear is applied at the extremity of a very long rod, a ſcratch made at the other end with a pin can be heard diſtinctly.

Thoſe phenomena have ſuggeſted a notion that ſound is conveyed by ſome ſubtle medium exiſting in all bodies: and if air is neceſſary for the communication of ſound, it is only in conſequence of its being replete with this ſubtle medium.

LI.

Flame is obſerved to be extinguiſhed in vacuo.

If a lighted candle is placed in a cloſe receiver filled with atmoſpheric air, it will be extinguiſhed after ſome time, although the air is not exhauſted.

LII.

Water deprived of the air inherent in it, will abſorb the atmoſpheric and ſome other kinds of air which communicate with it.

Water is deprived of its air by boiling or by remaining for a ſufficient time in an exhauſted receiver.

LIII.

Water evaporates continually in open air; in a cloſe veſſel the evaporation is greater, as the air is more exhauſted, for a certain period of time: evaporation is alſo increaſed by heat.

This experiment ſeems to diſprove the opinion which attributes the aſcent of vapour is to the attraction of the air.

ON THE EFFECTS OF HEAT IN ALTERING THE SPECIFIC GRAVITIES OF BODIES, AND ON THE MEASURES OF HEAT.

LIV.

Heat applied to a given body either ſolid or fluid, increaſes its magnitude, and conſequently diminiſhes its ſpecific gravity.

Thoſe bodies which are changed by heat from ſolid to fluid ſubſtances, are in many caſes exeptions to this article; for inſtance, water is ſpecifically heavier than ice; but bodies in general, ſuch as are the objects of philoſophical experiments, are obſerved to be increaſed in magnitude by heat.

In eſtimating ſpecific gravities, when great exactneſs is required, allowance is made for the effects of heat.

LV.

On the methods commonly uſed to eſtimate degrees of heat by the expanſion of a fluid.

Mercury, ſpirits of wine, and ſome oils, have been uſed for this purpoſe.

The heat of a fluid cannot be increaſed after it has began to boil. The temperature of water when juſt beginning to freeze, and that of boiling water, are ſtandards to which other degrees of heat are referred.

LVI.

If the magnitude of any quantity of mercury at the temperature of water juſt beginning to freeze, is denoted by unity, when it is heated to the temperature of boiling water, its magnitude will become = 1.0157.

In the conſtruction of the thermometer commonly uſed, this augmentation of the ſpace occupied by a given quantity of mercury, is divided into 180 equal parts or degrees, the initial point of which correſponds to the temperature of freezing, and is uſually denoted by the number 32; conſequently the degree anſwering to boiling water muſt be 212. It may perhaps ſeem that the ſpace anſwering to the number .0157 ſhould be divided geometrically inſtead of arithmetically; but ſuch a diviſion would be uſeleſs in practice: if the ratio of 1.0157 to 1, were divided into 180 equal ratios, the numbers reſulting would not differ materially from thoſe which ariſe from dividing .0157 into 180 equal parts, and adding unity to them. For example, the degree of heat correſponding to 80 in Fahrenheits thermometer would be denoted by a magnitude $= \overline{1.0157}^{\frac{48}{180}} = 1.00416278$ by the geometrical diviſion: by the arithmetical, the magnitude correſponding to the ſame degree will be found $= 1.00418667$ not differing the third part of a degree from the former.

As the magnitude of a fluid is always increaſed by increaſing its heat, the heights of the mercury in the thermometer when applied to different bodies will denote when one body is hotter than the other, and when they are of the ſame temperature. But it cannot be ſaid how much or in what proportion the heat of one body exceeds that of another, ſince there is no method of meaſuring the quantity of heat either by ſpace or number. If the expanſion or proportion of magnitude were the geometrical meaſure of heat, it would follow that a given increaſe of heat would expand all fluids in the ſame proportion, whereas there are few, if any, fluids which are equally expanded by the ſame increaſe of heat. The expanſion of a fluid has however been aſſumed as a geometrical meaſure of heat, but on conditions, which, ſuppoſing them true in any ſingle inſtance, are certainly not general.

LVII.

It being granted that heat is meaſurable by the expanſions of a given fluid, and that in equal ſmall inſtants of time equal degrees of heat are communicated to it, ſo that the increment of magnitude during that ſmall given time may be to the whole magnitude in a conſtant proportion; then it will follow, that if the times in which a body is heated, be taken in arithmetical progreſſion, the degrees of heat or magnitudes of the fluid in the thermometer correſponding will be in geometrical progreſſion.

Let the magnitude of the mercury in a thermometer at freezing point be a, let y be the increaſe of its magnitude by the application of any heat continued uniform for the time x: in any ſmall portion of time $\dot{x}$, let $\dot{y}$ be the increaſe of magnitude in the mercury correſponding, and becauſe in any ſmall portion of time $\dot{x}$, the magnitude of $a + y$ is always increaſed in a given proportion, $\dot{y}$ will be to $a + y$ in a given or conſtant proportion; let this conſtant proportion be that of $\dot{x} : t$ t being a definite invariable time to be determined afterwards; then ſince $\dot{y} : a + y : : \dot{x} : t$ we have $\dot{x} = t \times \frac{\dot{y}}{a+y}$ or $x = t \times \log. \frac{a+y}{a}$: from whence it immediately follows that if x repreſents quantities in arithmetical progreſſion, the magnitudes $a+y$ correſponding will be in geometrical progreſſion.

The conſtant quantity t is determined by one experiment: applying the uniform heat when the temperature is at the freezing point in the time x, let the increaſe of the mercury's magnitude be y, x and y being both obſerved by experiment, t will be $= \frac{x}{\log. \frac{a+y}{a}}$. Any other given temperature as well as that of the freezing point might be aſſumed to correſpond with the conſtant magnitude a. It is evident that the conditions on which this propoſition is founded are too precarious to admit of its being applied with any effect to practice.

LVIII.

Let warm water be placed under the receiver of an air-pump: If the air is exhauſted, the water is obſerved to boil at a leſs temperature than when it is heated in open air.

In the conſtruction of thermometers, allowance is made for the alteration in the heat of boilng water occaſioned by the different weights of the atmoſphere as indicated by the barometer.

LIX.

The heat of ſteam from boiling water, contained in a veſſel, covered from the external air, is obſerved to be the ſame with the heat of boiling water.

LX.

If a given quantity of air is ſhut up in a cloſe veſſel; the application of heat will increaſe its elaſtic force.

This elaſtic force may be meaſured by a barometer or ſimilar inſtrument.

LXI.

If air has liberty to expand freely, the application of heat will diminiſh its denſity.

Air at the temperature of freezing expands about .00244 parts of the whole for each degree of the thermometer when the preſſure is equivalent to 30 ½ inches of mercury.

The elaſtic force of air at liberty to expand freely in the atmoſphere is increaſed by heat, but is equally decreaſed by the diminution of the air's denſity; conſequently the elaſtic force of the heated air will the ſame as that of ſurrounding adjacent air: be the altitude of the mercury in the barometer will not be ſenſibly affected by any difference of the air's temperature at a given ſtation; the expanſion of the mercury itſelf not being here conſidered.

ON THE VELOCITIES OF FLUIDS WHICH FLOW FROM SMALL APERTURES.

LXII.

If water flows out of a cylindrical or any priſmatic ſmall aperture inſerted in a veſſel, the velocity of the effluent water will be equal to

to that which a heavy body acquires by falling freely from reſt through a ſpace which is equal to the diſtance between the aperture and the fluid's ſurface.

Suppoſe the water to flow through a cylinder the axis of which is horizontal; let the area of its baſe be a, the length $=b$, the diſtance of the aperture from the fluid's ſurface $=d$. $l=$ the ſpace through which gravity impels bodies falling freely from reſt in one ſecond The force which impels the water at the baſe of this cylinder is the weight of a column of water $=ad$, and the cylinder being conſidered full of water the maſs moved by the force ad will be ab, from whence is obtained the force which accelerates the effluent water $=\frac{d}{b}$. This force acts on the water at the baſe during its progreſs through the cylinder or through the ſpace b, and will at the end of that ſpace have generated a velocity $=\sqrt{\frac{4ld\times b}{b}}=\sqrt{4ld}$ which is the velocity acquired by a body which falls freely through a ſpace $=d$ or the perpendicular diſtance between the aperture and ſurface of the fluid. It appears that however ſmall the length of the cylinder may be, the ſame velocity is generated in the water which iſſues through it; wherefore, if it is evaneſcent or of the leaſt finite magnitude, the velocity of the water which flows from it will be the ſame as that which is determined in this ſolution, provided the water flows out in a cylindrical or priſmatic form, correſponding to the aperture through which it flows.

LXIII.

If water flows out of equal apertures made in the ſides of veſſels, kept conſtantly filled, it is obſerved that the quantity of water diſcharged in a given time, is in a direct ſubduplicate ratio of the perpendicular diſtances of the apertures from the ſurface of the water.

LXIV.

If an aperture biſects the altitude of a cylindrical veſſel, kept conſtantly filled with water, the horizontal diſtance to which the water ſpouts, is equal to the height of the veſſel, ſetting aſide the effects of friction, and the reſiſtance of the air.

LXV.

If apertures be made at different diſtances from the ſurface of the water contained in a cylindrical veſſel kept conſtantly filled, the horizontal diſtance to which the fluid ſpouts, is obſerved to be the greateſt when the aperture biſects the diſtance between the ſurface of the water and baſe of the veſſel.

Let A be the altitude of the veſſel, D the diſtance of the aperture from the ſurface: the parameter of the parabola deſcribed by the fluid will be 4D, the abſciſſa $= A - D$, and the ſquare of the ordinate, or of the horizontal diſtance is equal to $4AD - 4D^2$, which quantity is the greateſt when $A = 2D$, or when the aperture biſects the diſtance between the ſurface of the water, and the baſe of the veſſel.

LXVI.

Let apertures be made at equal diſtances from the bottom and top of a cylindrical veſſel kept conſtantly filled with water; the horizontal diſtances to which the water ſpouts from theſe apertures is obſerved to be the ſame.

ON THE MOTION OF BODIES IN FLUIDS.

LXVII.

A cylinder moving in a fluid in the direction of its axis is reſiſted by a force equal to the weight of a column of the fluid, the baſe of which is the baſe of the cylinder, and altitude equal to the ſpace through which a body muſt fall freely from reſt to acquire the velocity of the cylinder's motion.

The velocity of the cylinder is ſuppoſed ſuch that the ſpace deſcribed by it in its paſſage may be immediately filled by the ſurrounding fluid; which condition may be otherwiſe expreſſed by ſuppoſing the fluid to be infinitely compreſſed.

LXVIII.

A ſphere moving in a fluid is oppoſed by a reſiſtance which is to the force which reſiſts a cylinder moving in the direction of its axis with the ſame velocity, in the proportion of 1 to 2.

LXIX.

Small ſpherical bodies deſcend in fluids with velocities that are nearly uniform, and are in the ſubduplicate ratio of their diameters.

If the diameters are diminiſhed beyond a certain degree, the velocity of deſcent will be ſo ſmall as to be imperceptible, except after an interval of time.

LXX.

A ſphere of evaneſcent weight aſcends in any fluid with a velocity equal to that which a body acquires by falling freely from reſt through a ſpace equal to $\frac{4}{3}$ of the ſphere's diameter.

The aerial bubbles that riſe in fluids during the ſolution of metals, &c. are of this kind; theſe ſpherules being of no ſenſible weight. It is evident from the article that the velocities with which the ſpheres aſcend are in a ſubduplicate ratio of their diameters.

LXXI.

Spheres of evaneſcent weight and of a given diameter riſe with the ſame velocities in fluids of any different denſities.

When the denſities of the fluids in which ſpheres without weight aſcend are altered, the motive forces by which the ſpheres are impelled, vary in the ſame proportion with the forces of reſiſtance.

LXXII.

Let ſpherules of evaneſcent weight conſiſting of an elaſtic medium aſcend in a fluid: if the preſſure of the fluid is diminiſhed, the velocities of aſcent will be increaſed.

The reaſon is this: the preſſure on the ſpherules being diminiſhed, their magnitudes and conſequently their diameters will be increaſed; and their velocities will be alſo increaſed in a ſubduplicate proportion of the diameters. When the elaſtic force of the medium contained in the ſpherules is inverſely as the ſpace it occupies, and the ſpherules aſcend in air, if the denſity of the air is diminiſhed in the proportion of $n : 1$, the velocities of the ſphere's aſcent will be increaſed in the proportion of $1 : n^{\frac{1}{6}}$.

LXXIII.

The velocities with which elaſtic ſpheres of evaneſcent weight aſcend in a fluid, are increaſed by heat and diminiſhed by cold.

Heat augments the capacities of the ſpheres and conſequently their diameters: the velocities will be increaſed in a ſubduplicate ratio of their diameters.

LXXIV.

A hollow ſphere formed of a ſubſtance the ſpecific gravity of which is to that of the atmoſphere as n to 1, will, when exhauſted,

hauſted, juſt float in the air, if its thickneſs is equal to its diameter divided by $6n$.

There is no neceſſity to aſſign limits to this approximation, as the value of n correſponding to any ſubſtance of which a hollow ſphere can be formed, is not leſs than 700 or 800.

LXXV.

Every thing elſe remaining as in the laſt article, let the hollow ſphere be filled with a fluid of which the ſpecific gravity is m, when that of common air is $= 1$: the ſphere ſo filled will float in the atmoſphere if its thickneſs is equal to its diameter multiplied into $\frac{1 - m}{6n}$.

LXXVI.

A hollow ſphere filled with inflammable air will float in the atmoſphere, if its weight in averdupoiſe ounces when empty, is equal to the cube of the number of feet in its diameter, multiplied into the number .589049.

The ſpecific gravity of air is here aſſumed at $\frac{1}{800}$ when that of water is 1; the ſpecific gravity of inflammable air being $\frac{1}{8000}$.

ELEC-

ELECTRICITY.

ARTICLE I.

IF a glaſs tube, made very dry, be rubbed with a piece of ſilk, it is obſerved that light ſubſtances, ſuch as feathers, bran, cork balls, gold leaf, &c. being applied near to it, are firſt attracted and afterwards repelled.

The power by which theſe light ſubſtances are attracted and repelled, is called Electricity; the ſubſtance, by the friction of which it is produced, is ſaid to be electric; the friction of electric bodies, is called excitation.

II.

Excited ſealing-wax being applied near to light bodies, firſt attracts and afterwards repels them.

Amber, ſilk, jet, dry wood, and a variety of other ſubſtances being excited, attract and repel light bodies, and are called Electrics.—Metals, water, and other bodies, the friction of which will not produce this power of attraction and repulſion, are called Non-electrics.

III.

If a metallic cylinder be ſupported upon ſilk lines, or upon glaſs, and an excited electric be applied near to it, every part of the cylinder will attract and repel light ſubſtances, in the ſame manner as the excited electric itſelf.

Metals poſſeſſing the power of tranſmitting electricity, are called Conductors; to theſe may be added charcoal, water, and all bodies which contain it. Green wood is a partial conductor of electricity by means of the water it contains; if the water is expelled by heat, the wood becomes a non-conductor.

IV.

Let a ſilk line, or a glaſs rod made very dry, be ſuſpended on glaſs; it is obſerved that electricity cannot be tranſmitted through either of them.

Glaſs being impervious to electricity is called a Non-conductor. Silk, jet, ſealing-wax, air, &c. are likewiſe Non-conductors; and in general, all bodies which are electrics, are Non-conductors; and all bodies which are Non-electrics, are Conductors.

A body which communicates with nothing but electrics, is ſaid to be inſulated.

V.

Let two cork balls be ſuſpended from lines about ſix inches in length, and inſulated; if excited ſealing wax or glaſs be applied to them, they will repel each other; and after the excited electric is removed, they are obſerved to remain repelled.

The uſe of inſulating bodies, is to confine the electricity applied to them, and to render the effects of it permanent.

VI.

Conducting ſubſtances not inſulated, being held near to an excited electric, are obſerved to be attracted towards it.

Definition.—The electric power which is produced from the excitation of glaſs, is called the Vitreous Electricity; and the power which is produced from the excitation of ſealing-wax or reſin is called the Reſinous Electricity.

VI.

If the inſulated cork balls repel each other with the vitreous electricity, the reſinous power being applied to them, will deſtroy that repulſion, and will cauſe the balls to repel each other with the reſinous electricity, if the excitation be ſufficiently ſtrong.

VIII.

The repulſion of the balls electrified with the reſinous power, is deſtroyed by the application of the vitreous.

IX.

If glaſs and ſealing-wax, equally excited, are at once applied to the inſulated cork balls, no electricity of either ſort will appear to be communicated to them.

We obſerve from the three preceding experiments, that the vitreous and reſinous powers counteract each other; if both are applied at once to a body, the electricity communicated upon the whole, will be only the difference of the two, and will be of that ſort which is the ſtrongeſt.

X.

If bodies be electrified with contrary powers, they are obſerved to attract each other ſtrongly.

General Properties of Electricity.

1. Bodies electrified vitreouſly repel each other.
2. Bodies electrified reſinouſly repel each other.
3. Electrified bodies attract bodies not electrified.
4. Bodies electrified by contrary powers attract each other ſtrongly.

XI.

If a glaſs tube, having its poliſh taken off, be excited, it will communicate the reſinous electricity to bodies held near it.

XII.

A piece of down being applied to excited ſealing-wax, and touched at the ſame time with a conductor, is electrified with the vitreous power.

It is concluded from this and the preceding experiment, that both reſinous and vitreous powers are produced from the excitation either of glaſs or of ſealing-wax.—For the ſake of diſtinction, that power which is produced by the excitation of poliſhed glaſs, will be called vitreous, and the contrary power reſinous.

XIII.

If the rubber of a glaſs tube or cylinder be inſulated, and a conductor applied to the cylinder while excited, it is obſerved that the rubber is electrified reſinouſly.

Whenever the electric powers are ſeparated by excitation, one power attaches itſelf to the excited electric, and the other to the rubber.

XIV.

If the rubber and conductor applied to an excited electric, are both inſulated, the leſs electricity will be produced, the more perfect the inſulation is made.

Neither the rubber nor conductor can be inſulated perfectly, on account of the moiſture which is always floating in the air.

If the air and the parts of the apparatus be very dry, little or no electricity will be excited in the circumſtances abovementioned.

It is concluded from what has preceded that the electric powers do not exiſt in the electrics themſelves, but that they are produced from the earth by the excitation of electrics.

An electric ſubſtance contained between parallel ſurfaces, however diſpoſed, is called an electric plate.

XV.

Electrified bodies attract bodies not electrified, although a thin electric plate be interpoſed between them.

XVI.

Bodies electrified with contrary powers, attract each other ſtrongly, although an electric plate be interpoſed between them.

Thus flame communicates heat through ſubſtances, which are impervious to the flame itſelf.

HYPOTHESIS.

1. The two electric powers exiſt together in all bodies.

2. Since they counteract each other when united, they can be made evident to the ſenſes, only by their ſeparation.

3. The two powers are ſeparated in nonelectricks by the excitation of electrics, or by the application of excited electrics.

4. The powers cannot be ſeparated in electric ſubſtances.

5. The two electricities attract each other ſtrongly through the ſubſtance of electrics.

6. Electric ſubſtances are impervious to the two electricities.

7. Either power, when applied to an unelectrified body, repels the power of the ſame ſort, and attracts the contrary power.

XVII.

If one ſurface of a plate be electrified with either power, the oppoſite ſurface is electrified with the contrary power, if it be not inſulated.

Either power being applied to one of the ſurfaces, attracts the contrary power through the ſubſtance of the electric, and repels the electricity of the ſame ſort with itſelf.—The two powers being thus brought to the oppoſite ſurfaces of the plate which is impervious to them, remain ſuſpended; ſtrongly attracting each other, till the interpoſed plate is broken by their force, or a communication is formed between the two ſurfaces by ſome conducting ſubſtance.

When the two powers are ſuſpended on the oppoſite ſurfaces of an electric plate, the plate is ſaid to be charged. A conducting ſubſtance which forms a communication between two charged ſurfaces is called a circuit.

XVIII.

When the two powers unite by breaking a plate of air, an electric light is obſerved, which is attended with an exploſion if the charge be ſufficiently great; the union of the two electricities deſtroys the effects of each, and leaves the plate diſcharged.

Whenever either power is accumulated in an electric cloud, the ſurface of the earth oppoſed to it is electrified by the contrary power.—Theſe powers ſometimes unite by breaking the interpoſed plate of air.

XIX.

If plates of air and glaſs equal in magnitude and dimenſion, are charged by equal excitations, the plate of air is obſerved to be much more eaſily broken by the force of the charge, than the plate of glaſs.

XX.

If inſulated bodies be applied to either ſurface of a charged plate, at different diſtances from it, it is obſerved that the leſs the diſtance, the ſtronger the repulſive power. If bodies not inſulated are applied, the leſs the diſtance from the charged ſurface, the ſtronger is the power of attraction; from whence it is concluded, that the forces of electric repulſion and attraction vary in ſome inverſe ratio of the diſtance.

It appears that, every thing elſe being the ſame, a thin electric plate of glaſs will explode with greater force than a thicker plate, the attraction of the two powers being the ſtronger, the nearer the charged ſurfaces are to each other.

XXI.

The force of an electric charge is obſerved not to depend upon the ſhape in which the charged ſurfaces of a given magnitude and thickneſs are diſpoſed.

When the ſurfaces of an electric plate are charged with a certain quantity of the two powers, it is obſerved that no additional electricity can be communicated to them, however great an excitation is applied for that purpoſe.

It may be inferred from the preceding experiments, that the force of a plate fully charged depends upon two things:

1. The quantity of the ſurfaces charged.
2. Their proximity.

XXII.

If any part of the human body forms a portion of the electric circuit, it is obſerved that the diſcharge is perceived in that part only which forms the communication, unleſs the charged ſurfaces are very great.

XXIII.

If circuits of different lengths, conſiſting of different conducting ſubſtances, form communications between the two charged ſurfaces of an electric plate, the diſcharge will, under certain limitations, be made through the beſt conductor, whatever be the lengths of the others.

If one circuit conſiſts of undried wood, and is of conſiderable length in compariſon of another which conſiſts of metal, the diſcharge will be made wholly through the latter, unleſs the charge ſhould be very great; in which caſe ſome ſmall part of the charge will paſs through the wood.

If a ſhort metallic rod and any part of the human body form two circuits between the ſame charged ſurfaces, the diſcharge will, in general, be made wholly through the metallic rod; but if the charge is very great, or if the metallic rod is very ſlender, or if it ſhould be of conſiderable length, in any of theſe caſes, the diſcharge may be perceived to paſs through that part of the body which forms one of the circuits.

XXIV.

If circuits of the ſame ſubſtance be different in length, the diſcharge will, in general, be made through the ſhorteſt of them

This will be the caſe when the charge is ſmall; but it may be ſo increaſed as to paſs through both the longer and ſhorter circuits.

XXV.

The times of diſcharging the ſurfaces of an electric plate through the longeſt or ſhorteſt circuits are not ſenſibly different.

Many miles of the earth's ſurface, large tracts of water, &c. have been made a part of the electric circuit, and the diſcharge through them has been inſtantaneous.

XXVI.

If either ſurface of a charged plate communicate with the earth, the power on the oppoſite ſurface will expand itſelf into a conductor which is contiguous to it, although the conductor be inſulated.

XXVII.

XXVII.

Let one ſurface of a charged plate be inſulated: although the other communicates with the earth, no diſcharge of either ſurface will follow.

XXVIII.

Let the ſurfaces of an electric plate be very ſlightly charged and inſulated, and let a circuit with ſucceſſive interruptions be formed; the two powers will be viſible, illuminating the points of the interrupted circuit, and each power will appear to extend further from the ſurface contiguous to it, the ſtronger charge is communicated to the plate; but if the illuminations on each ſide meet, there will immediately follow an exploſion of the whole charge.

The length of the interrupted circuit uſed for this experiment was 12 feet.

A cylindrical plate is that which is contained between two equal circles, the planes of which are parallel to each other.

XXIX.

If a cylindrical plate of air contained in the receiver of an air-pump be charged, it is obſerved that the more air is exhauſted from between the ſurfaces, the more eaſily the two powers will unite.

The diſcharge appears to be made chiefly from that ſurface which poſſeſſes the vitreous power.

XXX.

If an exhauſted receiver be made part of the electric circuit, and the charge ſhould be not ſufficient to cauſe an exploſion, an electric light will appear to proceed in oppoſite directions from the parts communicating with the vitreous and reſinous ſurfaces.

If two contiguous clouds, very high in the atmoſphere, be electrified with contrary powers, their diſcharge cauſes an appearance ſimilar to the diffuſe light obſerved in the receiver.

XXXI.

The ſtrength of the excitation being the ſame, it is obſerved that a charged plate of air will be the more eaſily broken, the ſmaller quantity of air is interpoſed between the oppoſite ſurfaces.

An electric plate is more eaſily diſcharged through an exhauſted receiver than in the open air; for the more rare the air is made by exhauſtion, the leſs reſiſtance will be oppoſed to the union of the two powers poſſeſſing the oppoſite ſurfaces, and attracting each other through the electric.

XXXII.

XXXII

The leaſt cylinder, the diameter of which is finite, and the thickneſs evaneſcent, is to the leaſt cylinder, the thickneſs of which is finite, and the diameter evaneſcent, in a greater than any aſſignable ratio.

XXXIII.

The electric powers poſſeſſing the oppoſite ſurfaces of a cylindrical plate of air, the diameter of which is evaneſcent, will be incomparably leſs reſiſted by the interpoſed electric, than by the thinneſt plate of air contained between two ſurfaces of a finite magnitude.

XXXIV.

If an interruption is made in each of two ſimilar circuits, which form communications between the charged ſurfaces of an electric plate, and if the ſpace of air in one of the interruptions is terminated by points, and in the other by balls, the diſcharge will be made through the circuit of which the points make a part, although the length of the interrupted ſpace of air is conſiderably greater than that in the other circuit.

Before any diſcharge takes place, the two powers are ſuſpended on the oppoſite ſurfaces of the charged Electric.

An electric plate may be diſcharged two ways: either ſilently in ſome ſenſible portion of time, or by exploſion in an inſtant: in either caſe, experiments abundantly ſhew that, cæteris paribus, the diſcharge will be made through a pointed body in preference to a round termination.

When a pointed body is preſented to any charged ſurface, a cylindrical plate of air, of evaneſcent diameter, is charged with the contrary electricities ſtrongly attracting each other through it; and the quantity of air being ſo ſmall, there will be little reſiſtance to their union: the diſcharge will be made by exploſion in preference to the gradual diſcharge, according as the oppoſite ſurfaces (the pointed body and the ſurface oppoſed to it) are larger, as they are nearer each other, and as the charge is greater: for it will be obſerved that a point, or very ſmall ſpherical termination which is in a phyſical ſenſe a point, will diſcharge any quantity of electricity ſilently and gradually without exploſion, while it is at a ſufficient diſtance from the oppoſite charged ſurface: by bringing it nearer, the method of diſcharge will be altered; which will now be by a ſucceſſion of ſmall exploſions very quickly following each other. The reaſon of this ſeems to be that when the charged ſurfaces are very near, there is not ſufficient time for the contrary powers to unite gradually, nor ſufficient room in which they may be diffuſed among the ſurrounding air.

This is confirmed by again removing the two oppoſed ſurfaces to ſuch a diſtance that the diſcharge may be made gradually; in this caſe, if the parts of the apparatus are ſo diſpoſed by any kind of contrivance, that the diſcharge muſt neceſſarily be made ſuddenly, the method of diſcharge will be again altered, becoming now a ſucceſſion of exploſions inſtead of a gradual current between the oppoſed ſurfaces: this ſuddenneſs in the diſcharge may be effected by a proper uſe of interruptions in the circuit: it may alſo be cauſed by motion; if either ſurface be moved briſkly toward the other, the exploſion of the diſcharge will be promoted.

Elevated conductors applied to buildings as a ſecurity from the effects of lightning, will contribute to diſcharge the electricity from a cloud that paſſes over them; and a greater quantity of the diſcharge will paſs through a pointed conductor, than through one which is terminated by a ball; but whether the diſcharge will be made by a gradual current, or by exploſion, will depend on the ſuddenneſs of the diſcharge, on the proximity of the cloud, its motion, and the quantity of the electricity contained in it. If a ſmall cloud hangs ſuſpended under a large cloud loaded with electric matter, pointed conductors on a building underneath will receive the diſcharge by exploſion in preference to thoſe terminated by balls, the ſmall clond forming an interruption which allows only an inſtant of time for the diſcharge. If a ſingle electric cloud is driven with conſiderable velocity near to a pointed conductor, the charge may be cauſed to explode upon it by the motion of the charged body. In other caſes, pointed conductors contribute to diſcharge a thunder cloud gradually without exploſion.

Mr. Wilſon's experiments, publiſhed in the Philoſophical Tranſactions 1778, have contributed greatly to explain the effects of points in diſcharging the electric matter.

If a conical pointed body were inſerted into a ſimilar hollow cone, formed in an electrified ſolid, the ſurfaces of the two cones being equi-diſtant, no greater diſcharge of the electricities would follow, than if the two conical ſurfaces had been plane, and oppoſed to each other at the ſame diſtance.

In order to increaſe the force of the charge, a number of electric plates are ſometimes joined together—the uſual method is, to form a communication between the inſides of a number of coated glaſs jars, and another communication between the external coating. Jars ſo diſpoſed are called a battery, the effects of which are ſimilar to thoſe of a ſingle plate of the ſame thickneſs with the jars, and containing the ſame quantity of ſurface.

XXXV.

If a large battery be diſcharged through a ſlender piece of wire, the wire will be broken to pieces or melted.

XXXVI.

If a battery be diſcharged through a quire of paper, a perforation is made through it, and each of the leaves is protruded by the ſtroke from the middle toward the outward leaves.

XXXVII.

Let a quire of paper be ſuſpended by a line in the manner of a pendulum from any convenient altitude, ſo that its plane may be vertical: let the largeſt charge from a battery be cauſed to paſs through it while quieſcent, in an horizontal direction perpendicular to the plane, the rods of communication not touching the paper; the phenomena are: 1ſt, the aperture mentioned in the laſt article, the leaves being protruded both ways from the middle: 2nd, not the ſmalleſt motion is communicated to the paper from the force of the diſcharge.

A quire of the thickeſt and ſtrongeſt paper was made uſe of for this experiment, the height from which it was ſuſpended being 16 feet. It is an extraordinary appearance, on the hypotheſis of a ſingle electric fluid, that a force ſufficient to penetrate a ſolid ſubſtance of great tenacity and coheſive force ſhould not communicate the ſmalleſt motion to the paper, when a breath of air would cauſe ſome ſenſible vibration in it. But this difficulty is not unanſwerable: for a velocity may be aſſigned, with which a body impinges againſt and paſſes through a pendulum of any given weight and reſiſting force, ſo that a ſmaller angular velocity ſhall be communicated to it than any that may be propoſed; and we know no limit to the velocity of the electric power or powers. But the other phenomenon, i. e. the oppoſite direction in which the leaves are protruded, tends very much to ſtrengthen the opinion of two oppoſite currents: perhaps either of thoſe phenomena, conſidered ſingly, may admit of eaſy ſolution from the hypotheſis of a ſingle power; when they are taken both together, it ſeems more difficult to reconcile this hypotheſis with matter of fact.

XXXVIII.

If two electric plates be charged, and a communication formed between the vitreous ſide of one, and the reſinous ſide of the other, no diſcharge will follow; unleſs a communication be formed between the other two ſurfaces, at the ſame time.

The natural electricity in the atmoſphere is frequently diſcharged in this manner: Two clouds being electrified with oppoſite powers, the ſurfaces of the earth immediately under them are likewiſe electrified with powers contrary to thoſe in the clouds above them; and the moiſture of earth forming a communication between the two contiguous charged ſurfaces, whenever the two clouds meet, there will follow a diſcharge, both of the clouds and ſurfaces on the earth oppoſed to them.—If the earth ſhould be dry, and conſequently afford a reſiſtance to the union of the two electricities accumulated on or under its ſurface, there will follow an exploſion in the earth as well as in the atmoſphere, which will produce concuſſions and other phænomena which have frequently been obſerved to happen in dry ſeaſons, particularly in thoſe climates which are the moſt liable to ſtorms of thunder and lightning.

MAGNETISM.

ARTICLE I.

LET a ſlender iron rod exactly balanced, be ſuſpended on a point ſo as to revolve freely in a plane parallel to the horizon: If the rod be touched with with a loadſtone, one extremity of it will be always directed towards the north.

That power which the loadſtone communicates to iron is called Magnetiſm; a rod of iron poſſeſſed of this power is called a Magnet: Magnetiſm is communicated to iron by magnetical rods, as well as by the loadſtone itſelf: Iron rods alſo become magnetical by friction, by being kept for a long time in the ſame poſition, and by electricity, without any aſſiſtance from the loadſtone or magnets. The magnetical needle does not point exactly to the north, but varies from that direction by different angles; at ſome places pointing to the eaſt, at others to the weſt of the meridian: at the ſame place the magnetic needle is obſerved to change its azimuth gradually: in England the motion of its direction is weſtward, being at preſent between 22° and 23° weſt from the north point of the horizon. The extremities of a magnet are called Poles. A vertical circle in the Heavens, which interſects the horizon in the points to which the magnetical needle when at reſt is directed, is called the Magnetic Meridian.

II.

If the north pole of a magnet be placed at the point of ſuſpenſion of a needle, and drawn along towards either extremity; that part of the needle which is touched by the magnet will be directed to the ſouth.

III.

If the ſouth pole of a magnet be placed to the point of ſuſpenſion of a needle, and drawn along toward either extremity; that part of the needle which is touched will be directed to the north.

IV.

Either pole of a magnet is obſerved to attract iron, which is not magnetical.

V.

The north poles of two magnets repel each other when brought contiguous.

VI.

The ſouth poles of two magnets repel each other when brought contiguous.

VII.

The contrary poles of two magnets attract each other ſtrongly.

VIII.

The northern magnetiſm is obſerved to be deſtroyed by the communication of the ſouthern, and *vice verſa*.

The two ſorts of magnetiſm operate in ſuch a manner as to counteract each other; if both be communicated to the arm of a needle, the effect upon the whole will be only the difference of the two, and will be of that ſort which is ſtrongeſt.

From this principle it is eaſy to reverſe the poles of the magnet.

IX.

Let a needle be exactly balanced and ſuſpended, ſo as to revolve freely in a vertical plane; if it be made magnetical when horizontal, the north pole will be depreſſed, and the ſouth pole elevated above the horizon.

X.

The diſcharge of the electric powers through a needle communicates magnetiſm to it, if the charge be ſufficient.

General Properties of Magnetiſm.

1. No ſubſtance is capable of communicating or receiving magnetiſm, except the loadſtone and iron.
2. Every magnet has two poles.
3. Similar poles of magnets repel each other.
4. Either pole attracts iron not magnetical.
5. The contrary poles of magnets attract each other ſtrongly.
6. The properties of electricity and magnetiſm are in ſome reſpects analogous to each other. Vid. Electricity, Art. X. and XVI.

OPTICS.

Article I.

RAYS of light proceed from luminous bodies in right lines, and in all directions.

It is generally agreed, that light conſiſts of very minute particles of matter, which are ſent forth by ſome power, the cauſe of which is unknown, from luminous bodies, in right lines, and in all directions.

Single particles of light, ſucceeding each other in a right line, conſtitute a ray of light, conſidered in a mathematical ſenſe; but, phyſically ſpeaking, a ray is the leaſt viſible portion of light ſeparated from the reſt.

A ray of light being turned out of its courſe by falling upon a poliſhed ſubſtance, through which it does not paſs, is ſaid to be reflected.

The angle contained between the incident ray and a line drawn perpendicular to the ſurface from the point whereon the ray falls, is called the angle of incidence.

The angle contained between the perpendicular and the reflected ray, is called the angle of reflection.

II.

The incident and reflected rays are in the ſame plane, and the angles of incidence and reflection are equal.

The rays do not impinge upon the reflecting ſurface, being turned out of their courſe before they arrive at it, by ſome power which is contiguous to the ſurface, and acts in a direction perpendicular to it.

III.

III.

Rays of light proceed from illumined bodies in right lines and in all directions.

The ſurfaces of thoſe bodies which conſiſt of uneven and irregular parts reflect the light in all directions.

A ray of light being turned out of its courſe by falling obliquely upon the ſurface of a tranſparent medium which it paſſes through, is ſaid to be refracted; the angle contained between the incident ray and a perpendicular to the ſurface drawn from the point whereon the ray falls, is called the angle of incidence; the angle contained between the refracted ray and the perpendicular above-mentioned is called the angle of refraction.

The difference of the angles of incidence and refraction, is the angle by which the ray deviates from its original direction, and is called the angle of deviation.

Rays of light are refracted by ſome power contiguous to the refracting ſurface.

Rays of light being bent out of their courſe by paſſing near the ſharp edges of bodies, are ſaid to be inflected.

IV.

When a ray falls obliquely upon the ſurface of a denſer medium, it is refracted towards the perpendicular; a ray, falling obliquely upon the ſurface of a rarer medium, and paſſing through it, is refracted from the perpendicular.

A ray, falling perpendicularly upon a refracting ſurface, is not turned out of its courſe, but proceeds in the ſame direction with the incident ray.—While a ray moves in the ſame uniform medium, it does not change its direction.

The principles of Optics are demonſtrated upon a ſuppoſition that light is an homogeneal ſubſtance; and though light will appear to be compounded of ſeveral kinds of rays, yet the principles of refraction and reflection, &c. are mathematically true when applied to rays of any one ſort.

V.

When a ray of light is refracted out of air into a given medium, or out of a given medium into air, the ſine of the angle of incidence is to the ſine of the angle of the refraction in a given ratio.

If a ray of light is refracted out of air into glaſs, the ſine of incidence is to the ſine of refraction as 31 is to 20, or as 3 to 2 nearly; out of air into water, as 4 to 3. Hence the ſine of incidence is to the ſine of refraction, when a ray paſſes out of water into glaſs, as $\frac{3}{4} : \frac{20}{31}$, or as 93 : 80.

VI.

A ray of light cannot paſs out of a denſer medium into a rarer, if the angle of incidence exceeds a certain limit.

This limit is the angle of which the ſine is to radius as the ſine of incidence is to the ſine of refraction out of the denſer medium into the rarer. Thus a ray of light will not paſs out of water into air if the angle of incidence exceeds 48° 36′, the ſine of which is $\frac{3}{4}$ of radius.

A ray of light will not paſs out of glaſs into air if the angle of incidence exceeds 40° 11′, the ſine of which is $\frac{20}{31}$ parts of radius; or out of glaſs into water, if the angle of incidence exceeds about 59° 20′.

VII.

VII.

When a ray of light is refracted by paſſing out of a rarer medium into a denſer, if the angle of incidence be increaſed, the angle of deviation will alſo be increaſed.

Let i, r, and d, repreſent the leaſt contemporary variations of the angles of incidence, refraction, and deviation; we have from the properties of thoſe angles $i = r + d$ and $d = i - r$; but the leaſt increment of incidence is to the contemporary increment of refraction as the tangent of incidence to the tangent of refraction; hence i is greater than r; therefore d being poſitive, the angle of deviation will of courſe increaſe while the angle of incidence increaſes.

The ſame reaſoning may be applied to demonſtrate, that when a ray of light is refracted by paſſing out of a denſer medium into a rarer, the angle of incidence being increaſed, the angle of deviation will alſo be increaſed.

A priſm is a ſolid terminated by three rectangular parallelograms and two equal and parallel triangles.

A line drawn through the priſm parallel to the interſections of the parallelograms is called an axis of the priſm.

VIII.

A ray of light paſſing through a denſer priſm, perpendicular to the axis, is obſerved to be refracted towards the thicker part of the priſm.

The angle contained between the ſides of the priſm through which a ray paſſes, is called the refracting angle.

It is always ſuppoſed, in optical experiments, that when a ray is refracted through a priſm, the incident and emergent rays are in a plane which is perpendicular to the axis.

IX.

If the angles of incidence in which a ray of light impinges upon the ſurfaces of a glaſs priſm be ſmall, the deviation of the ray from its original courſe, after refraction, will equal about half of the refracting angle.

Two angles being diminiſhed *ſine limite*, may vaniſh in any given ratio, and their ſines will vaniſh in the ſame ratio. This is a mathematical propoſition; but in practice, although two angles be not evaneſcent, yet if they exceed not a few minutes, the ratios of the arcs and of the ſines are ſo nearly the ſame, that, as far as regards experiments depending upon them, no ſenſible difference can be obſerved, whether the ratio of the arcs or that of the ſines be aſſumed: thus if the two arcs are $8'$ and $4'$, the ſines are as 23271 to 11636 nearly; which differs very little from the ratio of 8 : 4.

Cor. I. In general let $I =$ the angle of incidence, $R =$ the angle of refraction, out of the rarer medium into the denſer, and $D =$ the angle of deviation after the two refractions, $V =$ the refracting angle of the priſm; then, if the angles of incidence be very ſmall,

$$D = \frac{V \times \overline{I - R}}{R}:$$

in the ſame priſm, therefore, the angle of deviation will remain the ſame, however the angles of incidence upon the ſurfaces (being always ſmall) ſhould vary.

Cor. II. In different priſms formed of the ſame ſubſtance, the angle of deviation D will be proportional to the refracting angles of the priſms.

A double convex lens is a ſolid contained between two ſegments of ſpherical ſurfaces.

A line joining the centres of the ſpherical ſurfaces, is called the axis of the lens.

X.

There is a point in every double convex lens, through which if a ray paſſes, the incident and emergent rays will be parallel, however the incident ray be inclined to the axis of the lens; this point is called the centre of the lens.

If tangents be drawn to the lens, at the points of incidence and emergence in any given plane, theſe tangents will be parallel.

The lens being very thin, or if the direction of the ray's paſſage be not much inclined to the axis, the incident and emergent rays will be nearly in the ſame ſtrait line.

It is underſtood that the lenſes, priſms, &c. uſed in optical experiments, are formed of tranſparent ſubſtances which are denſer than the medium through which the rays flow before they arrive at the refracting ſurfaces. Theſe ſubſtances are uſually glaſs or water.

XI.

A ray of light falling upon a convex lens is bent by refraction towards the axis, unleſs the inclination of the incident ray to the axis be very great.

A ray of light falling upon any curve ſurface is refracted or reflected in the ſame manner as if it had fallen upon a plane ſurface which touched the curve in the point of incidence: for which reaſon it is the ſame thing, whether a ray is refracted through a lens or through a priſm,

the

the ſides of which touch the lens in the points through which the ray paſſes.

XII.

If a ray of light falling nearly parallel to the axis be refracted through a glaſs lens, it will deviate the more from its original direction the more remote from the centre, it falls upon the lens.

If a priſm be conceived to touch the lens in the points through which the ray paſſes, the angle at the vertex of the priſm will be the greater, the more remote thoſe points are from the axis of the lens; and conſequently, by Art. IX. the angle of deviation will be the greater.

XIII.

If a ray of light be refracted through a medium, terminated by parallel ſurfaces, the incident and emergent rays are parallel.

The ſurfaces at the points of incidence and emergence are here ſuppoſed parallel.

A ſlender portion of rays, ſeparated from the reſt, is called a pencil of rays.

Pencils of rays are uſually cylindrical or conical; the axis of a pencil is the ſame as the axis of the cone or cylinder.

XIV.

Parallel rays falling on a reflecting plane ſurface are reflected parallel.

XV.

If a pencil of parallel rays falls perpendicularly upon a ſpherical reflector, the focus of reflected rays biſects the diſtance between the ſurface and centre.

Foci are thoſe points from which rays of light diverge, or to which they converge.

If parallel rays falling upon a reflecting or refracting ſurface are cauſed to diverge from, or to converge to a given point, this point is called the principal focus.

The diſtance between the principal focus and the ſurface of the refractor or reflector, is called the principal focal length.

A ſpherical reflector is ſuppoſed to be a ſegment of a ſpherical ſurface.

A line drawn through the centres of the ſphere, and of the leſſer circle which terminates the reflector, is called the axis.

In the determination of focal lengths, the axis of the pencil of incident rays is ſuppoſed to paſs through the centres of the refracting or reflecting ſurfaces, and the mathematical concluſions which follow are only true of thoſe rays which are contiguous to the axes; but experiment admits of a conſiderable deviation from the limitations abovementioned.

XVI.

If diverging or converging rays fall upon a ſpherical reflecting ſurface, the diſtance of the focus of incident rays from the principal focus, half the radius of the reflector, and the diſtance between the principal focus and focus of the reflected rays, will be in continual proportion.

Suppoſe the reflector to be concave, and the rays to diverge from a focus, the diſtance of which from the ſurface $= d$.

Let the radius of the reflector $= r$; we have by the preceding rule, $d - \frac{r}{2} : \frac{r}{2} :: \frac{r}{2} : \frac{r^2}{4d-2r} =$ the diſtance between the focus of reflected rays and the principal focus.

The focus of reflected rays is in this caſe between the principal focus and the centre of the reflector, wherefore adding $\frac{r}{2}$ to the quantity laſt found, $\frac{r^2}{4d-2r} + \frac{r}{2}$ $= \frac{2r^2 + 4dr - 2r^2}{8d-4r} = \frac{dr}{2d-r}$ will be the diſtance of the focus of reflected rays from the ſurface.

This ſolution extends to all the caſes of foci formed by reflection from a ſpherical ſurface, by changing the ſign of r when the reflector is convex, and of d when the rays converge to a point, the diſtance of which from the ſurface is d; thus, if rays converge upon a concave reflector, the radius of which is 30 inches, and focus of converging rays ſhould be 10 inches from the ſurface, the focal length required will be $\frac{-dr}{-2d-r} = \frac{dr}{2d+r}$ in the preſent caſe $= \frac{30 \times 10}{30+20} = 6$.

The diſtance between the ſurface of the reflector, and the focus of reflected rays, is called the focal length.

XVII.

The foci of incident and reflected rays are always on the ſame ſide of the principal focus.

If the focus of incident rays be moved along the axis of the reflector, the focus of reflected rays will move in the oppoſite direction, and the foci will meet at the ſurface and centre.

XVIII.

Parallel rays paſſing through one or more plane refracting ſurfaces, any how inclined, emerge parallel.

XIX.

The diſtance of the centre of a glaſs ſphere, from the principal focus of rays refracted by it, is nearly equal to a radius and a half of the ſphere.

1. In general let $r =$ the radius of the ſphere, the ſines of inſidence and refraction I and R, the diſtance of the principal focus from the centre of the ſphere $=$

$$\frac{Ir}{2I - 2R}.$$

2. Parallel rays paſſing through a globe of water, are collected into a focus, the diſtance of which from the centre is equal to a diameter of the ſphere.

3. If a pencil of parallel rays falls upon a ſpherical refracting ſurface, we have the following proportion: as the difference of the ſines of incidence and refraction

is to the ſine of incidence, ſo is the radius of the ſurface, to the diſtance between the ſurface and focus of refracted rays; therefore, I and R, ſignifying as uſual, and r being the radius of the convex ſurface of a denſer medium, the principal focal length will be $= \frac{Ir}{I-R}$.

XX.

A pencil of parallel rays falling upon a double convex glaſs lens, terminated by ſurfaces, the radii of which are equal, is obſerved to be collected into a focus, the diſtance of which from the lens is nearly equal to the radius of either ſurface.

If the radii of the ſurfaces be unequal, and the ſines of incidence and refraction I and R, we have the following rule.

Find the principal focal length after refraction from the firſt ſurface, and it will be, as the diſtance between the centres of the ſurfaces to the radius of the ſecond ſurface; ſo is the difference between the principal focal length before found, and the radius of the firſt ſurface, to the focal length required.

Suppoſe the lens to be double convex;
Let the radius of the firſt ſurface $=r$
the radius of the ſecond ſurface $=\varrho$

Then the principal focal length of the firſt ſurface $= \frac{Ir}{I-R}$, and we have from the proportion before mentioned, $r+\varrho:\varrho::\frac{Ir}{I-R}-r::\frac{Rr\varrho}{\overline{I-R}\times\overline{r+\varrho}}=$ the principal focal length of the lens, This ſolution extends to the principal focal lengths of all lenſes, the thickneſs of which is inconſiderable, the ſigns of r and ϱ being changed when the ſurfaces are concave inſtead of convex; if either ſurface is plane, the radius which belongs to it becomes infinite, in which caſe, the finite quantities added to, or ſubtracted from it, vaniſh.

In glaſs the principal focal length $= \frac{20r\varrho}{11r+11\varrho}$ or $\frac{2r\varrho}{r+\varrho}$ nearly: when $r = \varrho$ as in the experiment, the principal focal length $= \frac{2r^2}{2r} = r$.

Suppoſe the ſecond ſurface of a glaſs lens to become plane; ϱ then is infinite, and $f = \frac{2\varrho r}{r+\varrho} = \frac{2r\varrho}{\varrho} = 2r$, or, the focal length is twice the radius.

The principal focal length is the ſame, whether one or the other ſurface is oppoſed to the incident rays.

XXI.

Let $d =$ the diſtance between the focus of incident rays and the centre of a lens, $r =$ the principal local length, f the diſtance between the focus of refracted rays and the lens, and we have the following proportion: as $d - r : r$, ſo is d to f.

If the two radii of a glaſs lens be r and ϱ, it appeared from the laſt note that the principal focal length will be $\frac{20r\varrho}{11r+11\varrho}$, or $\frac{2r\varrho}{r+\varrho}$, ſufficiently accurate for moſt experiments. Let d = the diſtance of a focus of incident rays, f the diſtance of the focus of emergent rays from the lens, and we have the following proportion: as $d - \frac{2r\rho}{r+\rho} : \frac{2r\rho}{r+\rho} :: d : f$ wherefore $f =$

$$\frac{2dr\rho}{dr + d\rho - 2r\rho}.$$

Every thing elſe remaining, the focal length will be the ſame, whichever ſurface of the lens be oppoſed to the incident rays.

If the lens ſhould be of any ſubſtance in which the ſines of incidence and refraction are as I : R, inſtead of $\frac{2r\rho}{r+\rho}$, we muſt ſubſtitute the general expreſſion

$$\frac{Rr\rho}{\overline{I-R} \times \overline{r+\rho}},$$ and the proportion will become

$$d - \frac{Rr\rho}{\overline{I-R} \times \overline{r+\rho}} : \frac{Rr\rho}{\overline{I-R} \times \overline{r+\rho}} : d : f,$$ whence

$$f = \frac{Rdr\rho}{\overline{I-R} \times \overline{dr + d\rho} - Rr\rho},$$ which is a ſolution for all caſes of focal lengths when the thickneſs of the lens is inconſiderable.

It is next to be conſidered in what manner the combination of the foci of ſeparate pencils forms images or pictures of the objects from which the rays flow.

XXII.

Images formed by reflection from a plane ſurface, are erect, ſimilar and equal to the objects, and are as far diſtant behind the ſurface as the objects are before it.

XXIII.

XXIII.

An object being immerſed in water, appears nearer to the eye than it really is by $\frac{1}{4}$ of its depth from the water's ſurface.

In general, if rays diverge from an object, and fall upon a refracting ſurface, out of a denſer medium into a rarer, the diſtance of image will be to the diſtance of the object from the refracting ſurface, as the ſine of incidence to the ſine of refraction, out of the denſer medium into the rarer.

XXIV.

If an object be the arc of a circle, having the ſame centre with a ſpherical reflector, the image will be a ſimilar concentric arc; and the magnitudes of the image and object will be to each other in the ſame proportion as their diſtances from the centre of the reflector.

Having given $r =$ radius of a concave reflector, $d =$ the diſtance of the object from the ſurface, we have the diſtance of the image $= \frac{dr}{2d-r}$, and the magnitudes of the object and the image, as $2d-r : r$.

The diſtances of the object and image from the centre of the reflector are in the ſame proportion as their diſtances from the ſurface.

XXV.

Every thing remaining as in the laſt article, the image is obſerved to be inverted, or erect, with reſpect to the object, according as they are on different ſides or on the ſame ſide of the centre of the reflector.

If the object is moved along the axis of the reflector, the image will be moved in a contrary direction.

XXVI.

An object ſeen by reflection from a ſpherical convex ſurface, appears diminiſhed.

The apparent magnitude depends on the angle which the image ſubtends at the eye; in the preſent caſe, the image being nearer to the centre than the object, will be leſs than the object in the ſame proportion.

XXVII.

If the arc of a circle, the centre of which coincides with the centre of a double convex lens, be conſidered as an object, the image will be an arc ſimilar to the object, and the magnitudes of the object and image will be to each other, as their diſtances from the centre of the lens.

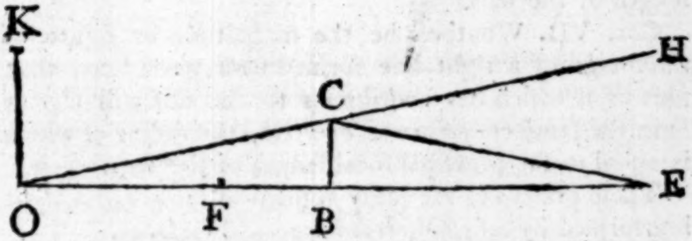
The properties (with regard to focal length) of all lenſes, the principal focal length of which is given, are the ſame. Wherefore, let r be the principal focal length, d the diſtance of an object from the lens;

then, whatever be the radii of the ſurfaces, the diſtance of the image from the lens $= \frac{dr}{d-r}$.

Every thing elſe remaining, no difference is obſerved in the focal length, which of the ſurfaces is oppoſed to the incident rays.

The magnitude, poſition, and diſtance of the image from the lens remain the ſame, whatever be the figure of the aperture through which the rays flow.—The brightneſs of the image will be as the area of the aperture when the magnitude of the image is given.

The image of a right line formed by a lens is not rectilinear, but coincides with one of the conic ſections, which is determined by the following conſtruction.



Let B repreſent the centre, OE the axis and FB the principal focal length of a double convex lens; let OK be a right line drawn perpendicular to the axis, conſidered as an object.

Draw BC=BF, at right rngles to BO: Join CO and produce it; at the point C deſcribe the angle HCE =BCO. B and E will be the foci of the conic ſection which coincides with the image, the major axis of which $=\overline{BC+CE}$.

Cor. I. The point F being being between B and O, the ſection is an ellipſe.

Cor. II. If O coincides with F, OCB and ECH will be each 45 degrees, and BCE a right angle, wherefore CE will be parallel to BE, or E wſll be at an infinite diſtance. The conic ſection is in this caſe a parabola.

U 2

Cor. III. If OB is leſs than BC, BCE will become greater than a right angle, and conſequently E will now be removed to the oppoſite ſide of B, in which caſe the ſection will be an hyperbola; theſe properties follow from the properties of conic ſections and of focal lengths.

Cor. IV. Suppoſe $OB = d$, $BC = p$, we have likewiſe from the properties of conics, the ſemiaxis major of the ſection $= \frac{d^2p}{d^2-p^2}$ and the ſemiaxis minor $= \frac{dp}{\sqrt{d^2-p^2}}$.

Cor. V. The principal parameter of the ſection is always equal to twice the principal focal length of the lens.

Cor. VI. The image of a very diſtant right line is a circle, the radius of which is equal to the principal focal length of the lens.

Cor. VII. Whatever be the magnitude or figure of the image of a right line formed by a given lens, that part of it which lies contiguous to the axis will always have the ſame curvature as a circle, the radius of which is equal to the principal focal length of the lens.

Theſe principles are eaſily applied to images of a right line formed by reflection from ſpherical ſurfaces.

XXVIII.

Objects ſeen through a double concave lens appear diminiſhed.

The apparent magnitude of an object, ſeen through lenſes of all kinds, is determined by the angle which the image ſubtends at the eye.

The apparent magnitude of the image depends on two circumſtances: 1ſt, the real magnitude; and 2dly, the diſtance of the eye from it, ; from whence we may derive an eſtimation of the apparent magnitudes of objects, ſeen through any given lens.

The image being ſmall when compared with its diſtance from the eye, will ſubtend an angle at the eye, which is as the length of the image directly, and the diſtance of the eye from it inverſely.

Let $d =$ the diſtance of an object from a double concave lens, $r =$ the principal focal length; the diſtance of the image from the lens will be $\frac{dr}{d+r}$ on the ſame ſide with the object; let $e =$ the diſtance of the eye from the lens; then according to the preceding principles, the angle ſubtended by the object, when ſeen by the naked eye, will be to that which it appears under, when viewed through the lens, as $\frac{d}{d+e}$ to $\frac{dr}{d+r}$ divided by $e + \frac{dr}{d+r}$, or as $dr+er+ed : dr+er$.

It is evident by inſpection that the latter quantity muſt be always ſmaller than the former; for which reaſon the object will appear diminiſhed, unleſs the lens be contiguous to the object or to the eye.

XXVIII.

An object ſeen through a double convex lens will appear magnified when the diſtance of the object from the lens is leſs than the principal focal length, at whatever diſtance the eye is ſituated from the lens.

Let $e =$ the diſtance of the eye from the lens, $d =$ the diſtance of the object from the lens, $r =$ the principal focal length; the angle, ſubtended by the object ſeen by the naked eye, will be to the angle under which it appears when viewed through the lens, as $re+dr-ed$ to $re+dr$; the latter of theſe quantities being always greater than the former, the object muſt appear magnified while e and d retain any finite values; thus, if the principal focal length of a reading glaſs be 12 inches, the diſtance of the glaſs from an object ſeen through it $= 4$ inches, and from the eye $= 6$ inches, the object will appear magnified in the proportion of $12 \times 6 + 4 \times 12$ to $12 \times 6 + 4 \times 12 - 4 \times 6$, or in the proportion of 5 to 4.

XXIX.

If the diſtance of an object from a lens, through which it is ſeen, be greater than the principal focal length, it will appear magnified if the eye is between the image and the lens.

The notation of the preceding article remaining, the angle under which the object is ſeen by the naked eye, will be to that under which it is ſeen through the lens, in the ſame proportion as before, or as $er + dr - ed$ to $er + dr$; from this proportion a ſingular phenomenon may be accounted for, which is this; if an object is placed in the principal focus of a lens, the apparent magnitude will continually be increaſed, *ſine limite*, as the eye recedes from the lens: for, referring to the preceding expreſſion, if $d = r$ the proportion will become that of r to $r + e$, the latter quantity always encreaſing with e, r remaining unaltered.

XXX.

If the diſtance of an object from a lens be greater than the principal focal length, and the eye is ſituated further from the lens than the image, the object viewed through the lens will always appear magnified, except when de is greater than $2er+2dr$, in which caſe it will appear diminiſhed.

When d, the diſtance between the object and the lens, is very great, the object will appear magnified if de is leſs than $2dr$, that is, if e is leſs than $2r$; if, therefore, the Sun or any other of the heavenly bodies be viewed through a lens, and the diſtance of the eye from the lens is leſs than twice the principal focal length, the object will appear magnified; but if the eye ſhould be at a greater diſtance from the lens, the object will be diminiſhed when ſeen through the lens.

XXXI.

When rays flow from an object through a lens, the image and object will be erect or inverted in reſpect of each other, according as they are on the ſame or different ſides of the lens.

All real images muſt therefore be inverted in reſpect of the objects.

If r repreſents the principal focal length of a lens, d its diſtance from the object, the lineal magnitude of the image will be to that of the object as r to $d-r$; if $d=2r$, the image and the object will be equal.

XXXII.

Let an object be ſituated at any given diſtance from a ſkreen, on which its image is to be repreſented by refraction of the rays through a lens; ſuppoſe the lens to be moved gradually from the object to the ſkreen, it will either paſs through two poſitions,

ſitions, in which it will diſtinctly repreſent the image of the object on the ſkreen, or there will be no poſition in which the lens will repreſent the image of the object.

It frequently happens that the analytical reſolutions of philoſophical, as well as other kinds of problems, produce more concluſions or anſwers, than were required or even known to exiſt. The experiment deſcribed in the article affords an eaſy and obvious example of this kind: the ſolution is as follows:

Suppoſe the given diſtance between the object and the ſkreen to be a, and it is required to find the diſtance x, at which a lens of which the principal focal length $=r$, muſt be placed from the object, ſo that the image may be diſtinctly repreſented on the ſkreen. Here, becauſe the diſtance between the object and lens is x, and the focal length of the lens $=r$, it follows that the diſtance between the lens and ſkreen on which the image is formed, will be $\frac{xr}{x-r}$; wherefore $x+\frac{xr}{x-r}=a$, and $x=\frac{a\pm\sqrt{a^2-4ra}}{2}$.

We here obtain not only an anſwer to the problem required, but further diſcover that there are two values of x, which will ſatisfy the conditions of the problem, when any image can be formed: it appears alſo from the ſolution, that if a^2 is leſs than $4ra$, that is, if the diſtance of the ſkreen from the object is leſs than four times the principal focal length of the lens, the diſtance ſought will be impoſſible, ſo that there can be no poſition of the lens by which an image can be formed on the ſkreen. When the diſtance of the ſkreen from the object is equal to four times the principal focal length, the two poſitions of the lens and the two images correſponding coincide; in this caſe the lens biſects the diſtance between the object and ſkreen, and the image is equal to the object.

XXXIII.

XXXIII.

The angle under which a very diſtant object is ſeen, through a teleſcope conſiſting of two double convex lenſes, is to the angle, under which the ſame object is ſeen by the naked eye, as the principal focal length of the object glaſs is to the principal focal length of the eye glaſs.

In this ſort of teleſcope, the diſtance between the two lenſes is equal to the ſum of their principal focal lengths.

Let the diſtance of the eye from the object glaſs be e, the principal focal length of the object glaſs $=r$; then, by the note to Art. XXXII. the angle ſubtended by the object, when ſeen by the naked eye, will be to that under which it appears through the teleſcope, as $e-r : r$; that is, as the principal focal length of the eye glaſs to that of the object glaſs.

XXXIV.

A minute object, when ſeen through a lens of very ſmall principal focal length, appears magnified and diſtinct, if the object be placed in the principal focus.

The angle under which the object appears, will be to that which it ſubtends when ſeen by the naked eye, as the diſtance at which it is viewed by the naked eye diſtinctly, to the principal focal length of the lens.

If the principal focal length of the lens does not exceed $\frac{1}{50}$ of an inch; and the diſtance at which the eye can ſee diſtinctly to be about ſix inches; it follows that the lens will magnify above 300 times in diameter.

In this experiment, the rays flowing from any point in the object will emerge from the lens parallel, and being collected into a focus upon the retina of the eye will render the image diſtinct: the uſual ſtructure of the eye is ſuch as to collect rays, which fall upon it nearly parallel, into a focus upon the retina, whereon the images of objects from which the rays flow are diſtinctly formed. It ſometimes happens that the eye, by too great a degree of convexity, cauſes each pencil to converge to a focus before it arrives at the retina, and conſequently, the rays which flow from each point of an object, are not collected into a correſponding point upon the retina, but are diſperſed over a ſmall circle, whereby the object is imperfectly repreſented upon the retina, and the ſmall circles above deſcribed, being intermixed, increaſe the confuſion of viſion. The application of a concave lens, properly adjuſted to the eye, removes the focus of each pencil further from the object, ſo as to fall exactly upon the retina.

A contrary defect ariſes when the eye becomes ſo flattened as to collect the rays into foci behind the retina, cauſing a confuſion of viſion ſimilar to the former. This is remedied by a convex lens, which aſſiſts the eye in collecting the rays, bringing the foci of the ſeveral pencils nearer to the object, and, by forming the image upon the retina, renders viſion diſtinct.

As to the ſtructure of the eye (the humours of which conſtitute a compound lens) and the uſes of its various parts; anatomical diſſection will convey more adequate ideas than can be derived from deſcription.

XXXV.

Small objects may be ſeen diſtinct and magnified through a combination of two or more lenſes.

Theſe combinations of lenſes are called compound microſcopes, of which there is a great variety of conſtructions.

XXXVI.

If a minute tranſparent object, illumined by the rays of the ſun, be placed before a ſmall lens, at a little greater diſtance from it than the principal focus, the image will be deſcribed on a ſkreen, properly placed, diſtinct and magnified.

If the diſtance of the object from the lens be d, and the principal focal length of the lens $= r$, the diſtance of the ſkreen or image from the lens will be $\frac{dr}{d-r}$, and the lineal magnitude of the object will be to that of the image as $d : \frac{dr}{d-r}$, or as $d-r : r$.

Upon this principle the ſolar microſcope is conſtructed.

XXXVII.

If the rays of the ſun are tranſmitted through a very ſmall aperture into a dark chamber, and are received perpendicularly on a ſkreen, they will form a round image of the ſun, equal to that which would be formed by the ſolar rays paſſing through a lens, the focal length of which is equal to the diſtance between the aperture and the image.

Suppoſe the aperture circular, and conſequently the pencils of rays which flow through it cylindrical.

The images which are formed by lenſes conſiſt of points, being the vertexes of cones or pyramids of rays. The image deſcribed in the article, conſiſts of the baſes of cylinders becauſe the rays which flow through the ſmall aperture are all cylindrical: but the aperture being in a phyſical ſenſe evaneſcent, the breadth of theſe cylinders will be evaneſcent, and conſequently they will have the effect of points in forming a well-defined image.

This experiment and the explanation of it will be preciſely the ſame, if a very ſmall aperture of any ſhape is uſed inſtead of that which is circular; the image always conſiſting of the baſes of priſms, which will approach more nearly to points the ſmaller the aperture which is made uſe of.

The ſolar rays are always ſuppoſed to paſs through the apertures in a direction perpendicular to their planes.

XXXVIII.

The diameter of the ſolar image is to its diſtance from the aperture, as twice the ſine of the ſun's apparent ſemidiameter to radius.

If the diſtance of the image from the aperture is 18 feet, the diameter of the ſolar image will be about two inches.

It muſt be remarked concerning the experiments on ſolar rays, that a ſpeculum is neceſſary for turning the rays out of their courſe into any direction which may be required.

The image formed by the ſolar rays which flow through a very ſmall aperture may be termed, for the ſake of diſtinction, the correct image, being of a given magnitude when the diſtance from the aperture is known, and better defined than when a larger aperture is uſed, which is neceſſary for the enſuing priſmatic experiments; more light being required for making them than is tranſmitted through a very ſmall aperture. The aperture uſed for priſmatic experiments is uſually about $\frac{1}{4}$ of an inch.

XXXIX.

When the ſolar rays flow through a circular aperture of any given magnitude, and are received perpendicularly on a ſkreen, a circular image will be formed, conſiſting of a bright central image, and an annulus of fainter light ſurrounding it.

Let a be the diameter of the correct image correſponding to the diſtance between the aperture and the ſkreen on which it is formed: let d be the diameter of the aperture: then the diameter of the bright central image will be $a - d$; the breadth of the annulus or penumbra will be d. Conſequently the diameter of the entire image will be $a + d$: and the difference between the diameters of the entire image and the bright central image will be $2d$.

It appears that when the diſtance from the aperture is given, the diameter of the bright central image is decreaſed as the diameter of the aperture is increaſed.

The internal parts of the penumbra, being thoſe which are neareſt to the centre of the image are almoſt of the ſame brightneſs with the central image itſelf, and are ſcarcely to be diſtinguiſhed from it: the other parts of the penumbra, which are more remote from the centre, decreaſe gradually in brightneſs to the extremity.

The diſtance from the aperture being given, and a being greater than d, the diameter of the bright central image is decreaſed as the diameter of the aperture is increaſed, until the diameter of the aperture is equal to that of the correct image, in which caſe the diameter of the bright central image vaniſhes. But the diameter of the aperture being ſtill increaſed, the diameter of the bright central image will continually increaſe alſo.

Similar changes take place when the diameter of the aperture remaining the ſame, the diſtance of the image from it and conſequently the diameter of the correct image is continually leſſened; the diameter of the bright image being diminiſhed at the ſame time till $a = d$, when the bright central image vaniſhes, and when a becomes leſs than d, the bright image will afterward increaſe from being $=0$ to its being $=$ to the aperture as the ſkreen is moved toward the aperture.

However the magnitude of the bright image may be changed, the breadth of the penumbra always remains the ſame while d is leſs than a.

XL.

When the ſolar rays flow through an aperture correſponding to any plane figure, the image which is formed will be determined thus; on the ſkreen which receives the ſolar rays perpendicularly, deſcribe a plane figure, ſimilar and equal to the aperture: let circles be deſcribed from centres coincident with the line or lines which terminate this figure, theſe circles will determine the figure of the image.

It is plain from this conſtruction, that the entire figure will not be ſimilar to the aperture in a geometrical ſenſe, although it will partake of the ſame form, and will approach more nearly to that of the aperture, the nearer the ſkreen is brought to it: as the ſkreen recedes from the aperture, the image approaches to a circular form with which it ultimately coincides if the diſtance is increaſed *ſine limite*. The penumbra will be a figure partly circular and partly ſimilar to the aperture, but will approach nearer to the figure of the aperture, the nearer the ſkreen is brought to it.

It is propoſed as a problem by Ariſtotle to explain the reaſon why the rays of the ſun flowing through a triangular aperture form a round image. From what has preceded, this problem ſeems to want limitation: becauſe when the ſkreen on which the image is received, is near the aperture, the image will be triangular; and at greater diſtances, the image always partakes ſomething of a triangular form, not being entirely round unleſs the diſtance of the image from the aperture is increaſed *ſine limite*.

XLI.

Whatever be the figure of an aperture through which the ſolar rays flow, and afterwards paſs through a lens; a round image of the ſun will be formed on a ſkreen placed at a proper diſtance, which will be preciſely equal to the correct image, the penumbra being removed by the lens.

Theſe articles relative to the ſun's image and penumbra are ſo applicable to the phenomena of the Camera Obſcura, that a few remarks on this ſubject may be here properly inſerted.

A ſmall aperture being made in the window ſhutter of a dark chamber, the adjacent objects, if they are ſufficiently illuminated, will be repreſented on a ſkreen by images ſimilar to the objects, but inverted in reſpect of them. The images are well defined when the aperture is ſmall and the objects are diſtant, becauſe in this caſe the pencils which flow from each point of the objects are nearly cylindrical, and have very ſmall baſes, conſequently each point of the object will be repreſented on a ſkreen by a ſmall circle; and the leſs this circle is, the more exactly will it repreſent the correſponding point. When a lens of proper focal length is applied, each of theſe ſmall circles is converted into a point which will render the images more diſtinct; but if the aperture is increaſed, or the objects brought nearer to it, in either caſe, the circles in the image which repreſent correſponding points in the object, become larger, and conſequently the images are more indiſtinct, but yet are ſufficiently defined to expreſs the objects, unleſs the aperture ſhould be increaſed beyond a certain limit, which will depend on the ſort of objects repreſented, the quantity of light, &c.

XLII.

A ray of the ſun's light being refracted through a glaſs priſm, is obſerved to be ſeparated into rays of different colours.

Theſe colours are red, orange, yellow, green, blue, indigo and violet, and are called the priſmatic colours from the experiment. The ſeparated rays deviate unequally from the courſe of the incident ray, whence they are ſaid to be differently refrangible. The refrangibility of the rays anſwers to the order in which the colours appear in the priſmatic ſpectrum, the red being the leaſt, and the violet rays the moſt refracted: either of the ſeven colours ſeparated from the reſt is called an homogeneal ray, or homogeneal light.

XLIII.

Homogeneal light cannot be further ſeparated or altered, as to its colour, by refraction through any number of priſms.

XLIV.

A ray of light, paſſing through an uniform medium, terminated by parallel ſurfaces, is not at all ſeparated into colours.

Whenever the homogeneal rays, after having been ſeparated from a common ray of light, are cauſed by ſubſequent refraction to emerge parallel, the emergent rays are colourleſs; although they may deviate from the courſe of the incident ray; in this experiment, the homogeneal rays emerge after the ſecond refraction parallel.

XLV.

Thoſe rays which are the moſt refrangible are alſo the moſt reflexible.

The ſine of incidence is to the ſine of refraction in the red rays, paſſing out of glaſs into air, as 50 : 77; wherefore, they will be reflected from the ſurface of glaſs, if the angle of incidence exceeds 40° 29′, the ſine of which is $\frac{50}{77}$ of radius; the ſine of incidence is to the ſine of refraction of the violet rays, as 50 : 78; thoſe rays, therefore, will be reflected if the angle of incidence exceeds 39° 52′, the ſine of which is $\frac{50}{78}$ of radius.

XLVI.

The homogeneal rays, being collected into a focus upon a ſkreen of white paper, are colourleſs.

XLVII.

If one of the homogeneal rays be ſtopped before it reaches the ſkreen, the ſpectrum

conſiſting of the remaining rays becomes coloured.

It appears from the preceding Article, that ſolar light is an hetorogeneal ſubſtance, conſiſting of ſeven ſorts of homogeneal rays—Let a ray of ſolar light be refracted out of air into any given medium; the homogeneal rays will impinge upon the refracting ſurface at a common angle of incidence, but the ſines of the angles in which the rays are refracted will be different; the angle contained between the red and violet rays, after refraction, is called the angle of diſſipation, and is the difference of the laſt angles in which thoſe rays are refracted.

Let the ſine of incidence be to the ſine of refraction of the rays of mean refrangibility, paſſing out of air into any medium, as $m : 1$, of the violet rays as $m + \dot{m} : 1$, and of the red as $m - \dot{m} : 1$.

The quantity $\dot{m}$ being conſtant in the ſame medium, is aſſumed as a meaſure of the diſperſing or diſſipating power. Thus the ſine of incidence is to the ſine of refraction in the rays of mean refrangibility, paſſing out of air into common glaſs, as $\frac{77.5}{50} : 1$, in the red rays as $\frac{77}{50} : 1$, and in the violet as $\frac{78}{50} : 1$, wherefore, putting $\frac{77.5}{50} = m$, we have $\frac{78}{50} = m + \dot{m}$ and $\dot{m} = \frac{1}{100}$. In the ſame manner, if the ſine of incidence be to the ſine of refraction of the rays of mean refrangibility, paſſing out of air into water, as $\frac{108.5}{81} : 1$, of the red rays as $\frac{108}{81} : 1$, and of the violet rays as $\frac{109}{81} : 1$: if w be made $= \frac{108.5}{81}$, we ſhall have $\dot{w} = \frac{1}{162}$, and $\dot{m} : \dot{w}$, or the ratio of the diſperſing powers exiſting in the two mediums, whereby they ſeparate the homogeneal rays, is that of $\frac{1}{100} : \frac{1}{162}$, or $81 : 50$.

Suppoſe that the ſine of incidence is to the ſine of refraction of red rays, paſſing out of air into flint glaſs, as $1.565 : 1$, of the violet as $1.595 : 1$: then the ratio of refraction in the rays of mean refrangibility is as

1.58 : 1. if $n = 1.58$, $\dot{n} = \frac{3}{200}$: and the ratio of the diſperſing powers of flint and common glaſs will be as $\frac{3}{200} : \frac{1}{100}$, or as 3 :2.

XLVIII.

A ray of ſolar light being refracted thro' the ſides of iſoſceles priſm of common glaſs, the refracting angle of which is 30°, when the rays of mean refrangibility paſs parallel to the baſe, the angle of diſſipation will be about 39′.

If a ray of ſolar light be refracted through one ſurface only, let $t =$ the tangent of refraction of the rays of mean refrangibility, $m : 1$ the ratio of refraction of the ſame rays, $\dot{m}$ the meaſure of diſſipating power; the angle of diſſipation will be ſubtended by an arc $= \frac{2\dot{m}t}{m}$, radius being 1.

Thus if a ſolar ray impinges upon a ſurface of common glaſs, at an angle of incidence $= 20°$, m being $= \frac{77.5}{50}$, or 1.55, the angle of refraction of the rays of mean refrangibility $= 12°.44' 52''$, and the arc which meaſures the angle of diſſipation $= \frac{2 \times \text{tang. } 12°.44'.52''}{100 \times 1.55} =$ $10'.3''$, radius being $= 1$.

If the rays are refracted through two ſurfaces inclined to each other, ſuch as the ſides of a priſm, let the ſine of the refracting angle $= a$, the coſine of the angle of refraction of the rays of mean refrangibility at the firſt ſurface $= p$, the coſine of refraction at the ſecond ſurface $= q$: then the angle of diſſipation, at which the violet and red rays are inclined to each other after the ſecond refraction $= \frac{2\dot{m}a}{pq}$. Thus in the experiment $\dot{m} = \frac{1}{100}$ $a =$ the ſine of 30°, $p =$ coſine 15°, $q =$ coſine 23°.39′.5″, and the arc which meaſures the angle of diſperſion (rad. being 1) $= \frac{2 \times \text{ſine } 30}{100 \times \text{coſ. } 15° \times \text{coſ. } 23°.39'.5''}, = 38'.51''$.

If a ray be refracted through parallel ſurfaces, the angle of diſſipation vaniſhes, becauſe *a*, or the ſine of the refracting angle, is in that caſe nothing.

XLIX.

Let the vertex of a flint glaſs priſm, the refracting angle of which = 23°.40′, be applied to the baſe of a common glaſs priſm, the refracting angle of which = 25°; a ray of ſolar light will paſs directly through the priſms, when their ſurfaces are contiguous, but the emergent ray will be coloured.

It was formerly imagined, that a ray of ſolar light would, in all caſes, emerge colourleſs after refraction, if it did not deviate from the courſe of the incident ray, the contrary to which, however, appears from this experiment invented by Mr. Dollond.

The ray is ſuppoſed to fall perpendicularly upon the ſurface of the priſm, the refracting angle of which is the greateſt.

The poſition of the priſms in the experiment is ſuch, that the effects of refraction upon the paralleliſm of the homogeneal rays paſſing through them are contrary to each other, and conſequently if they were equal, the rays would emerge parallel: but the flint priſm by its greater diſſipating power more than counteracts the ſeparation of the rays cauſed by their paſſage through the firſt priſm, which = 38′½, and, inverting the order of the colours, cauſes the red and violet tays to emerge inclined to each other, at an angle of 12′½, which is ſufficiently great to produce a ſenſible tinge of the priſmatic colours in the emergent rays.

The difficulty which chiefly impeded the improvement of teleſcopes was, to refract a ray ſo, that while it ſhould deviate conſiderably from its original courſe, the diſperſion of the homogeneal rays might be counteracted, and that by this means they might all emerge parallel, and of courſe free from colour, which is not by any means to be effected, except by the combination of tranſparent ſubſtances, the refracting and diſſipating powers of which are different.

L.

Every thing remaining as in the laſt experiment, let the vertex of a common glaſs priſm, the refracting angle of which is 10°, be applied to the baſe of the flint priſm; if a ray of ſolar light paſſes through the three priſms, when their ſurfaces are contiguous, the emergent ray will deviate about 5°.37′ from the courſe of the incident ray, but will be colourleſs.

In this caſe the two common glaſs priſms refracting the ray in the ſame direction, cauſe it to deviate from the courſe of the incident ray about 5°.37′ more than the deviation in the contrary direction, ariſing from refraction through a flint priſm; but the latter by its greater diſſipating power, exactly counteracts the ſeparation of the rays occaſioned by refraction through the other two priſms, ſo that the homogeneal rays emerge at length parallel, and of courſe colourleſs.

In this experiment the rays of mean refrangibility emerge at an angle of refraction $= 16^{\circ}.57'$. If a ſolar ray impinged upon the ſurface of the priſm laſt applied, at an angle of incidence $= 16^{\circ}.57'$, the angle of diſſipation, after emergence into air, would be $12'\frac{1}{4}$.—It appeared from the laſt experiment, that the diſſipation of the rays emerging from the two priſms $= 12'\frac{1}{4}$; for which reaſon (and on account of the contrary poſition of the priſms) the red and violet rays emerging inclined to each other at an angle of $12'\frac{1}{4}$ from the two priſms, and falling upon the ſurface of the third, will be refracted out of it colourleſs.

LI.

If a ray of ſolar light paſſes directly through any two priſms, the diſſiparing powers of which are unequal, it will be ſeparated into the homogeneal rays.

Although the deviations of the ray, after refraction at each priſm, be equal and contrary, yet one of the priſms poſſeſſing a greater power of diſſipating the ray than the other, and conſequently the emergent ray becomes coloured.

The aperture of a lens is terminated by the circumference of a circle, the centre of which coincides with the axis of the lens; the circumference abovementioned is always ſuppoſed to be contiguous to the lens, and conſequently its plane is perpendicular to the axis.—The annulus of the lens, which is more remote from the axis than the circumference which terminates the aperture, is ſuppoſed to be covered with an opake ſubſtance. If a point, or in practice, any ſmall luminous body ſituated in the axis of the lens, be conſidered as an object, the image which is formed by the rays flowing from it

through the leaſt aperture, ſufficient for viſion, is called the principal image.

If all the aperture be covered with an opake ſubſtance, except an annulus adjacent to its circumference, the image formed by rays paſſing through this annulus, is called the extreme image.

LII.

If homogeneal rays flow from an object placed in the axis of a double convex glaſs lens, the extreme image of the object will be nearer to the lens than the principal image.

Hence it aopears, that the rays which diverge from a given point in object, are not all collected into a point in the correſponding image, many images of the point being diffuſed over the ſpace contained between the principal and extreme images.—The diſtance between the principal and extreme images, is called the ſpace of diffuſion.

This aberration of the rays from the geometrieal focus, ariſing from the ſpherical figure of the refracting ſurfaces, is one of the cauſes of indiſtinct viſion in refracted teleſcopes,, &c.

LIII.

The extreme image of an object, formed by a double convex lens, is coloured and indiſtinct, if the proportion of diameter of the aperture to the principal focal length exceeds a certain limit.

The colour and indiſtinctneſs which are here obſerved in the extreme image, ariſe from the different refrangibility of the homogeneal rays compoſing ſolar light, which emerges from the lens with too great a degree of obliquity, and becomes ſenſibly ſeparated into the coloured rays; theſe form many images of the ſame point, all of which at different diſtances from the lens.—This aberration cauſes an indiſtinctneſs even in the principal image, but in all others, formed by rays flowing through the lens more remote from the axis, the indiſtinctneſs is very much increaſed.

The two ſorts of aberration above deſcribed are the chief impediments to the perfection of teleſcopes.

The method uſed for obviating theſe difficulties will appear in the following experiments.

LIV.

An object glaſs may be compounded of three lenſes, whereof two are double convex, made of common glaſs, incloſing a double concave of flint glaſs, ſo that the extreme and principal images of objects formed by it ſhall coincide.

The concave glaſs being of greater refracting power than the other two, will diminiſh the angle of deviation of the extreme rays, more than of thoſe which are nearer to the axis; the refracted extreme rays will, therefore, interſect the axis further from the centre of the compound lens, than if it had paſſed through a double convex lens only, of the ſame principal focal length.

The radii of the lenſes abovementioned may be ſo adjuſted, that the ſpace of diffuſion ſhall vaniſh.

LV.

The refracting and diſſipating powers of the three lenſes, which conſtitute a compound object glaſs, being given, the radii of the ſurfaces may be ſo adjuſted to each other, that the extreme, principal, and the intermediate images of objects formed by it ſhall be diſtinct and colourleſs.

It appears from the preceding experiments, that the ſame combination of lenſes, not only reduces the ſpace of diffuſion into an inſenſible quantity, but at the ſame time refracts the extreme rays to the geometrical focus without ſeparating them into the primary colours, whereby the aberration ariſing from the different refrangibility of the rays, as well as from the ſpherical figure of the lens, is wholly removed. Object glaſſes, conſtructed upon theſe principles, have been called perfect lenſes.

A difficulty has ariſen in the conſtruction of perfect lenſes, viz. to determine the power of diſſipating the homogeneal rays in different refracting ſubſtances, which does not obſerve any law of the refracting power, denſity, or of any quantities formed out of them.

LVI.

Let a common glaſs priſm, the refracting angle of which = 30°, be applied contiguous to a priſm of flint glaſs, the refracting angle of which = 19°; let the vertices of the priſms be placed in oppoſite directions, a ſolar ray being refracted through them will deviate from the courſe of the incident ray, but will not be ſeparated into the coloured rays.

In this experiment, the angle of incidence upon the firſt ſurface of the common glaſs priſm being = 0, the angle of refraction at which the ray emerges from the flint priſm = 16°.31′¾, and the angle contained between the incident and emergent rays = 5°.31′¾.

If a ſolar ray is refracted through the common glaſs priſm only, the angle of incidence being = 0, the angle of refraction at which the middle rays emerge = 50°.48′.18″; wherefore the angle of diſſipation will be meaſured by an arc (radius being 1) =

$$\frac{2\times\text{ſine }30^{\circ}}{100\times\text{coſ. }50^{\circ}.48'.18''}=54'.24'',$$

by the rule in note to Art. XLVIII.

In like manner if a ſolar ray impinges upon the flint priſm at an angle of incidence = 16°.31′¾, it will emerge at an angle of refraction = 50°.48′.18″, and it is inferred from hence that the angle of diſſipation after emergence will be meaſured by an arc (radius being 1) =

$$\frac{3\times\text{ſine }19^{\circ}}{100\times\text{coſ. }50^{\circ}.48'.18''\times\text{coſ. }10^{\circ}.22'.26''}=54'.$$

The angles of diſſipation above determined differ only 24″, which being imperceptible in experiments, the angles are, phyſically ſpeaking, equal.

Thus it appears that the two priſms operate equally upon the paralleliſm of the homogeneal rays, paſſing through them in the directions above deſcribed; and theſe effects by the poſition of the priſms tending to correct each other, the homogeneal rays will emerge, after being refracted through them, parallel and colourleſs.

As to determining the powers of diſſipation and refraction exiſting in different refracting ſubſtances, the following method ſeems as eligible as any.

Suppoſe a ſolar ray to paſs through a priſm (the refracting and diſſipating powers of which are required) impinging perpendicularly upon the firſt ſurface. Let the angle of diſſipation and the angle of refraction of the middle rays be accurately meaſured, and let a = the meaſure of the angle of diſſipation, t = the tangent of refraction (radius being 1.) The angle of incidence upon

the

the ſecond ſurface being equal to the refracting angle of the priſm, and the angle of refraction being determined as above, we obtain the ratio of refraction: if this be as $n : 1$, then the meaſure of the diſſipating power $\dot{n} = \frac{an}{2t}$.

It is to be remarked that the preceding rules concerning the angle of diſſipation are mathematically true only when $\frac{\dot{n}}{n}$ is leſs than any aſſignable quantity; but in the practical application of them to ſuch refracting ſubſtances as are commonly uſed in optics (eſpecially when the angles of refraction are ſmall) they will not differ from the truth more than a few ſeconds of a degree, the effects of which minute error are inſenſible in experiment.

LVII.

Coloured ſubſtances reflect the homogeneal rays of light, which are of the ſame colour with themſelves, more copiouſly than the other rays.

A red ſubſtance held in green rays, if all other light be perfectly excluded, appears green, though very faintly; when held in the red rays, it appears of a very vivid red, which ſhews that the ſubſtance is of ſuch a nature, as to reflect the red rays much more copiouſly than the green.

LVIII.

Let a circle be divided into ſeven ſectors, proportional to the ſpaces occupied by the colours in the priſmatic ſpectrum, and let theſe ſectors be painted with the primary colours, according to their proper order and proportion; if the circle be turned ſwiftly in its own plane round the center, the whole will appear nearly white.

The artificial colours being leſs pure than the natural, the mixture of the ſeven colours in the experiment will not produce a perfect whiteneſs, but will approach nearer to it, the more care is taken to divide the circle in true proportions, and the more nearly the colours made uſe of approach thoſe which compoſe a ray of ſolar light.

LIX.

The heat produced by collecting the rays of the ſun into the principal focus of a concave mirror or double convex lens, is obſerved to be the more intenſe the greater the area of the mirror or lens, and the leſs that of the focus into which the rays are collected.

The intenſity of the heat produced by burning mirrors or lenſes, is proportional to the ſquares of their diameters directly, and the ſquares of their principal focal lengths inverſely.

The intenſity of the ſun's rays, collected into the principal focus of a concave mirror, will be the ſame as in the principal focus of a double convex lens of the ſame area, the principal focal length of which is one half the radius of the reflector.

LX.

Rays of the ſun, however condenſed, communicate no heat to an uniform medium through which they paſs.

LXI.

If a ray of light falls upon a glaſs ſphere, and emerges after one reflection and two refractions; the priſmatic colours will be moſt viſible in thoſe emergent rays, which are inclined to the incident rays at an angle of 17° 50′.

The violet rays will appear moſt copiouſly at the angle mentioned in the experiment, but the red rays will appear when the incident and emergent rays are inclined at an angle of about 19° .25′: from whence it follows, that the breadth of a rainbow, formed by one reflection and two refractions through ſpheres of glaſs, would appear by obſervation $=1^\circ .35' + 32' = 2^\circ 7'$.

It may be uſeful to ſubjoin a few notes concerning the rainbow.

1. The firſt rainbow is that which is moſt uſually ſeen in the heavens, and is formed by one reflection and two refractions of the ſun's rays falling upon drops of rain; the ſecond rainbow is formed by the rays falling upon drops of rain, and emerging after two refractions and two reflections. Theſe bows are never ſeen but when it rains and the ſun ſhines at the ſame time, the drops of rain and the ſun being, with regard to the ſpectator's horizon, in oppoſite parts of the heavens.

2. The apparent ſemi-diameter of the bow is equal to the apparent altitude of the higheſt point of the bow, together with the altitude of the ſun's centre above the horizon.

3. The apparent ſemi-diameter of the firſt rainbow is the difference between four times the angle of refraction and twice the angle of incidence of thoſe contiguous rays which fall upon the drops of rain, and emerge parallel after two refractions and one reflection; this angle, be-

longing to the red rays, is about 42° .2′; hence no primary rainbow can be obſerved unleſs the altitude of the ſun's centre above the horizon is leſs than 42° .2′. For the ſake of brevity, the rays are here conſidered as flowing from the ſun's centre only.

4. Contiguous rays, falling upon a refracting globe, emerge parallel after two refractions and one reflection, when the leaſt increment of the angle of incidence is to the contemporary increment of the angle of refraction as 2 : 1.

5. Hence the tangent of incidence is to the tangent of refraction as 2 : 1.

6. To find thoſe two angles, the ſines of which are in the proportion of 4 : 3, and tangents as 2 : 1.

Let x and $y =$ the coſines of incidence and refraction reſpectively, then the ſquares of their ſines will be $1-x^2$ and $1-y^2$, radius being unity, and the ſquares of their tangents will be $\frac{1-x^2}{x^2}$ and $\frac{1-y^2}{y^2}$: wherefore, by the conditions of the problem,

$$1-x^2 : 1-y^1 :: 16 : 9$$

and $$\frac{1-y^2}{y^2} : \frac{1-x^2}{x^2} :: 1 : 4,$$ and joining theſe ratios

$$\overline{x^2 \quad : \quad y^2 \quad :: \quad 4 : 9}$$

and $x^2 = \frac{4y^2}{9}$, wherefore $1-x^2 = \frac{9-4y^2}{9}$; and, referring to the firſt proportion, $\frac{9-4y^2}{9} : 1-y^2 :: 16 : 9$: multiplying extremes and means, $9-4y^2 = 16-16y^2$, $y^2 = \frac{7}{12}$ and $y = \sqrt{\frac{7}{12}} =$ the coſine of the angle of refraction; hence we determine the angle of refraction $= 40°.12′.11″$, and the angle of incidence (being that the ſine of which is to the ſine of 40°.12′.11″, as 4 : 3) $= 59^\circ\, 23'\, 28''$ which gives $\frac{1}{2}$ the angle contained between the incident and emergent rays, or $\frac{1}{2}$ the ſemi-diameter of the bow $= 2 \times \overline{40^\circ\, 12'\, 11''} - 59^\circ 23'\, 28'' = 21^\circ\, 0'\, 54''$; wherefore, the angle between the incident and emergent rays, or the apparent ſemi-diameter of the bow, formed by the red rays, $=$ $42^\circ\, 1'\, 48''$, or about $42^\circ\, 2'$.

In the ſame manner the apparent ſemi-diameter of the rainbow. formed by the violet rays, is found to be about $40^\circ\, 17'$, which being ſubtracted from $42^\circ\, 2'$, leaves $1^\circ\, 45'$ for the apparent breadth of the bow. This would be the breadth if the ſolar rays flowed from the centre of the Sun only; and ſince the apparent diameter of the Sun is about $32'$, the apparent breadth of the bow will be increaſed by $32'$, ſo that it will appear by obſervation $= 2^\circ\, 17'$.

The following rules will extend to determine the angles of incidence and refraction of contiguous homogeneal rays, emerging parallel from refracting ſpheres of any kind, the number of reflections within the ſpheres being in general $= n$.

1. The contiguous rays will emerge parallel after n reflections and two refractions, when the leaſt variation of the angle of incidence is to the contemporary variation of the angle of refraction as $n + 1 : 1$.

2. Suppoſe $n - 1 : 1$ as $1 : t$, hence the tangent of incidence muſt be to the tangent of refraction as $1 : t$; let the ſine of incidence be to the ſine of refraction as $1 : s$: to find the angles of incidence and refraction, call their coſines x and y, and we have from the conditions of the problem,

$1 - x^2 : 1 - y^2 :: 1 : s^2$ radius being unity,

and $\dfrac{1 - y^2}{y^2} : \dfrac{1 - x^2}{x^2} :: t^2 : 1$, joining theſe ratios,

$$\overline{}$$
$x^2 : y^2 :: t^2 : s^2$

and $x^2 = \dfrac{t^2 y^2}{s^2}$, or $1 - x^2 = \dfrac{s^2 - t^2 y^2}{s^2}$: ſubſtituting this value of $1-x^2$ in the firſt proportion, we have $\frac{s^2-t^2y^2}{s^2}$ $: 1-y^2 :: 1 : s^2$, wherefore $1-s^2=y^2-y^2t^2$, $y^2=\frac{1-s^2}{1-t^2}$, and $y=\sqrt{\frac{1-s^2}{1-t^2}}=$ the coſine of refraction in general. The ſine of refraction $=\sqrt{\frac{s^2-t^2}{1-t^2}}$, and conſequently the ſine of incidence $=\sqrt{\frac{s^2-t^2}{1-t^2}}\times\frac{1}{s}$.

The angle of incidence being determined $=$ I, and the angle of refraction $=$ R; let $p=90°$, $n=$ the number of reflections $+1$; then half the angle contained between the incident rays and thoſe which emerge contiguous and parallel will be $=\overline{n-2}\times p-n\mathrm{R}+\mathrm{I}$; which expreſſion anſwers to all values of n; obſerving only that when $n=1$ as in the primary rainbow, $-n\mathrm{R}+\mathrm{I}$ becomes $+n\mathrm{R}-\mathrm{I}$, becauſe the rays in this caſe meet on the contrary ſide of the ſphere.

ASTRO-

ASTRONOMY.

I. ASTRONOMY is a branch of natural knowledge, which relates to appearances in the heavens and the cauſes of them.

II. Plain Aſtronomy is that part of this ſcience which deſcribes the phenomena and their immediate cauſes only; Phyſical Aſtronomy demonſtrates the firſt known cauſes of the phenomena.

III. A ſpectator viewing the heavens, conceives himſelf to be ſituated in the centre of a ſphere, upon the concave ſurface of which all the heavenly bodies are diſpoſed.

On this concave ſurface ſeveral figures are ſuppoſed to be deſcribed, ſerving only as a means of referring to any known ſtar with greater facility: theſe figures are called Conſtellations. It appears that it is of no conſequence (as far as regards the apparent places of the ſtars) of what magnitude this imaginary ſphere is, always ſuppoſing the centre of it to coincide with the centre of view. Thus, in an artificial celeſtial globe, the angular poſition of the ſtars is ſuch, as, if viewed from the centre of the ſphere, would be ſimilar to that which is obſerved in the heavens.

It comes next in order to conſider of ſome properties of the ſphere immediately relating to Aſtronomy.

IV. A circle, revolving round any diameter as an axis of motion, will generate a ſolid figure, which is called a ſphere.

V. The center of the generating circle is the centre of the ſphere; and any right line paſſing through the centre, and terminated by the ſurface of the ſphere, is called a diameter.

VI. Any circle, the plane of which paſſes through the centre of the ſphere, divides it into two equal parts, and is called a great circle, being diſtinguiſhed thereby from other circles, the planes of which not paſſing through the centre, divide the ſphere unequally, and are called leſſer circles.

The angular diſtance of two points ſituated on the ſurface of the ſphere is meaſured by the arc of a great circle intercepted between them.

VII. A line drawn from the interſection of any two lines in a plane, and perpendicular to them, is perpendicular to the plane itſelf

Any two lines in a plane which are inclined to each other, being parallel to a given plane, the planes will be parallel.

Thus if any two lines inclined to each other in a plane be horizontal, the plane will be parallel to the horizon.

VIII. A diameter of a ſphere, drawn perpendicular to the plane of any great circle,

cle, is called the axis, and the extremities of the axis are called poles of the great circle.

1. Every point in the circumference of a great circle is 90° diſtant from the poles.

2. Any two great circles of a ſphere biſect each other, becauſe the interſection of their planes is a diameter common to them both, and every circle is biſected by its diameter.

IX. Great circles paſſing through the poles of a great circle are called its ſecondaries.

Any ſecondary biſects its great circle and all the leſſer circles parallel to it.

X. The inclination of two planes to each other, is the ſame as the inclination of two lines drawn from the ſame point in the common interſection of the planes, and perpendicular to it.

XI. The inclination of the planes of two great circles to each other, is meaſured by the arc of a ſecondary common to them both, and intercepted between the planes.

XII. The planes of ſecondaries are perpendicular to the planes of their great circles, becauſe the inclination of a ſecondary to its great circle is meaſured by a quadrant.

XIII. The inclination of the circumferences of two great circles is called a ſpherical angle, and is the ſame as the inclination of the planes of the circles.

The meaſure of a ſpherical angle is, therefore, an arc of a ſecondary, common to both circles, and intercepted between them.

XIV. Three arcs of a great circle, neither being greater than 180°, compoſe a ſpherical triangle, in which there are ſix quantities, viz. three ſides and three angles, any three of which being given, the reſt may be determined by the rules in Trigonometry.

As the angular poſition of objects ſeen from a given point, and ſituated in the ſame plane, is meaſured upon the circumference of a circle of any intermediate radius, the centre of which coincides with the centre of view; in like manner the angular poſition of objects ſituated in different planes is eſtimated, by referring them to the ſurface of a ſphere, the centre of which coincides with the centre of view.

In what manner the properties of the ſphere, above deſcribed, are applied to the determination of the angular poſition and motion of the heavenly bodies, will be next conſidered.

CONCERNING THE APPARENT PLACE AND MOTION OF THE HEAVENLY BODIES.

I. All the ſtars appear to revolve daily in the heavens, deſcribing circles the planes of which are parallel to each other.

II. Among the circles deſcribed by the heavenly bodies in their diurnal revolution, there is one only, the plane of which paſſes through the ſpectator's centre of view; this circle is called the equator.

The eye of the obſerver is ſuppoſed to be coincident with the centre of the ſphere, upon the concave ſurface of which all the heavenly bodies are diſpoſed; the equator therefore muſt be a great circle of the heavens.

All other circles deſcribed by ſtars ſituated out of the equator, are leſſer circles of the ſphere parallel to the equator, and are called parallels of declination.

III. If, one extremity of a line continuing fixed, the other be always directed to a ſtar revolving in a leſſer circle, the line will by its motion deſcribe the ſurface of a cone, the vertex of which coincides with the fixed extremity of the line abovementioned, and the circumference of the baſe with the leſſer circle, which the ſtar deſcribes parallel to the equator.

IV. Two points taken 90° from the equator, are called the poles of the equator, or poles of the world.

V. Secondaries to the equator are called circles of declination; of theſe, 24 which divide the equator into equal prrts, containing 15° each, are called hour circles.

VI. The declination of a ſtar is its angular diſtance from the equator; meaſured on a circle of declination paſſing through the ſtar.

Thus, in the middle of ſummer, the ſun's declination is 23° .28′ north; when the ſun is in the equator, his declination is 0.

VII. Stars appear to move with the leaſt velocity which are the moſt remote from the equator, where the velocity of their apparent motion is the greateſt.

The apparent velocity of a ſtar in its diurnal revolution is as the coſine of declination.

VIII. The Sun and ſome particular ſtars, hereafter to be mentioned, are obſerved to change their places among the fixed ſtars, and to perform their revolutions in various periodic times.

The fixed ſtars always preſerve the ſame angular diſtances from each other, whereby they are diſtinguiſhed from the Sun and planets, which continually change their places both with regard to each other and the fixed ſtars.

The proper motions which have been diſcovered in the ſtars, Arcturus, Capella, and ſome others, are not here conſidered.

IX. The Sun compleats his revolution among the fixed ſtars in 365 days, 6 hours, and about 9 minutes, deſcribing a great circle in the heavens, called the ecliptic.

X. The ecliptic is obſerved to be inclined to the equator at an angle of about 23° 28′.

The ecliptic and equator, being great circles, biſect each other; wherefore the points of interſection muſt be 180 degrees diſtant; theſe are called the equinoctial points, for reaſons hereafter to be given.

XI. The ecliptic is divided into twelve portions, called ſigns, each containing 30 degrees; the firſt point of Aries coincides with one of the equinoctial points, and the firſt of Libra with the other.

The twelve ſigns are as follow:

Aries, Taurus, Gemini, Cancer, Leo, Virgo,
♈ ♉ ♊ ♋ ♌ ♍
Libra, Scorpio, Sagittarius, Capricornus, Aquarius,
♎ ♏ ♐ ♑ ♒
Piſces.
♓

The apparent motion of bodies in the ſame direction with that of the Sun among the fixed ſtars, is called *motus in conſequentia*; and the motion in the contrary direction, *motus in antecedentia*.

XII. The right aſcenſion of a ſtar is meaſured by an arc of the equator, intercepted between the firſt point of Aries, and a circle of declination paſſing through the ſtar, reckoning according to the order of the ſigns.

Thus in the middle of ſummer the Sun's right aſcenſion is 90 degrees.

XIII. The longitude of a ſtar is meaſured by an arc of the ecliptic, intercepted between the firſt point of Aries, and a ſecondary to the ecliptic paſſing through the ſtar, reckoning according to the order of the ſigns.

Thus, in the middle of ſummer, the Sun's longitude is 90°; when the Sun enters Libra, the longitude is = 180°.

XIV. The latitude of a ſtar is its angular diſtance from the ecliptic, meaſured on a ſecondary to the ecliptic paſſing through the ſtar.

The latitude of the Sun is therefore nothing.

XV. A ſecondary, common to the ecliptic and the equator, is called the ſolſtitial colure: a ſecondary to the equator, paſſing through the equinoctial points, is called the equinoctial colure.

The inclination of the ecliptic to the equator is therefore meaſured by an arc of the ſtolſtitial colure intercepted between them, and this arc will be the meaſure of the Sun's greateſt declination = 23° 28′.

XVI. The horizon is a great circle in the heavens, the plane of which is perpendicular to a plumb line hanging freely and at reſt.

It is evident that the horizon with all the circles depending on it are moveable; every different ſpectator having a different horizon.

The pole of the horizon above us is called the Zenith, the other pole the Nadir.

XVII. Secondaries to the horizon are called vertical circles.

The altitude of a ſtar is meaſured by the arc of a vertical circle, intercepted between the ſtar and the horizon; the complement of the altitude to 90° is the ſtar's zenith diſtance.

XVIII. A ſecondary, common to the equator and the horizon, is called the meridian; the meridian, therefore, paſſes through the pole of the equator and the zenith.

The meridian is a vertical circle, and the inclination of the equator to the horizon is meaſured by an arc of the meridian intercepted between them.

XIX. The azimuth of a ſtar is meaſured by an arc of the horizon, intercepted between the meridian and a vertical circle which paſſes through the ſtar.

XX. A vertical circle, which cuts the meridian at right angles, is called the prime vertical.

The interſections of the meridian and prime vertical with the horizon are called the Cardinal points; or the north, eaſt, weſt, and ſouth; the whole horizon being divided into 32 equal parts, each being = 11° 15′, forms the mariner's compaſs.

XXI. The diurnal apparent motion of the ſtars is from eaſt to weſt.

When a ſtar is ſaid to move apparently from eaſt to weſt, it is meant that a vertical circle always paſſing through the ſtar continually changes its azimuth, leaving the eaſtward and approaching the weſtward parts of the horizon.

XXII. The direction in which the ſun moves in the ecliptic, among the fixed ſtars, is contrary to that of his apparent diurnal motion. The direction of the planet's motion among the fixed ſtars is moſt frequently obſerved to be *in conſequentia*, but ſometimes *in antecedentia*: the planets are alſo obſerved at times to be ſtationary.

XXIII. An arc of the meridian, intercepted between the zenith and the equator, is called the latitude of the place.

XXIV. An arc of the equator, intercepted between the meridians of two places, is called the difference of their longitude; or, if the longitude of one of them be referred to the meridian of the other, the arc above deſcribed is called the longitude.

Thus if the longitude of Rome be referred to the meridian of London it is 9°¾ eaſt.

When the centre of the Sun is upon the meridian of any place, it is there ſaid to be noon-day; which therefore will be at different times at different places; and ſince the time elapſed between the Sun's leaving and returning to the ſame meridian is 24 hours, during which a circle of declination paſſing through the Sun's centre will have deſcribed 360°, eſtimated on the equator; it follows that in one hour it will have deſcribed 15°; in one minute of time, 15′; and in one ſecond of time, 15″ of a degree, &c.

XXV. The tropics are two parallels of declination, the diſtance of which from the equator is equal to the Sun's greateſt declination.

The diſtance of the tropics from the equator is therefore = 23°.28′.

XXVI. The arctic and antarctic circles are two parallels of declination, the diſtance of which from the poles is equal to the diſtance of the tropics from the equator.

The arctic circle is 23°.28′. from the north pole, and the antarctic at the ſame diſtance from the ſouth pole.

XXVII. The apparent time of day is meaſured by the angle contained between the meridian of the place and a circle of declination paſſing through the Sun's centre, allowing one hour for 15°, and proportionally for minutes and ſeconds of a degree.

Table I. in the appendix contains the degrees of the equator expreſſed in time at the rate of 1^h for 15^o &c.

In Table II. hours, minutes and ſeconds are expreſſed in terms of degrees, minutes and ſeconds of the equator: by theſe tables any angular motion of a circle of declination may be converted into time, and any given time may be converted into a correſponding angle at the pole or are of the equator.

Thus let it be required to convert 83 .40′.33″.8 into time. Referring to Table I. we have,

$80^{o} = 5^{h} \quad 20^{m}$

$3^{o} = . \quad 12$

$40^{m} = . \quad 2 \quad 40^{s}$

$30^{s} = . \quad . \quad 2$

$3^{s} = . \quad . \quad 0.2$

$.8_{s} = . \quad . \quad 0.05$

$83^{c}.40'.33''.8 = 5^{h} \quad 34^{m} \quad 42^{s}.25$

To convert 5^h 34^m 42^s.25 into degrees of the equator by referring to Table II. we have,

$5^{h} = 75^{o} \quad 0' \quad 0''$

$30^{m} = 7 \quad 30 \quad 0$

$4^{m} = 1 \quad 0 \quad 0$

$40^{s} = . \quad 10 \quad 0$

$2^{s} = . \quad . \quad 30$

$.2s = . \quad . \quad 3$

$.05^{s} = . \quad . \quad .8$

$5^{h}.34^{m}.42^{s}.25 = 83 \quad 40 \quad 33.8$

ON SOME APPEARANCES WHICH DEPEND ON THE CIRCLES OF THE HEAVENS, AND THE POSITION OF THE OBSERVER IN RESPECT OF THEM.

I. In a given latitude no ſtar, the polar diſtance of which is leſs than the latitude of the place, will deſcend below the horizon.

II. In latitudes leſs than about 66° 32′, the Sun's centre can never be wholly above the horizon during twenty-four hours.

III. When the Sun's declination is greater than the diſtance of the zenith from the pole, the Sun's centre will not ſet below the horizon, or will not riſe above it, according as the declination is of the ſame, or of a contrary denomination to the latitude.

In this and the preceding article the effects of refraction are not conſidered.

IV. If an obſerver be ſituated under the equator, the plane of his horizon is perpendicular to the plane of the equator, and parallels of declination.

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The ſphere of the heavens in this poſition in reſpect of the obſerver's horizon is called a right ſphere, the equator and all the parallels of declination are biſected by the horizon, therefore the times of riſing and ſetting of any ſtar divide the time of the diurnal revolution into equal parts.

The poles of the equator coincide with the horizon in a right ſphere.

V. If an obſerver is ſituated at either pole, his horizon is coincident with the equator, and the parallels of declination are parallels to the horizon.

The ſphere of the heavens in this poſition, reſpecting the obſerver's horizon, is called a parallel ſphere. The Sun, when above the horizon, appears to revolve in a leſſer circle, parallel to the horizon, and at an altitude above it equal to his declination.

VI. In all other poſitions of the ſphere of the heavens, except thoſe deſcribed, the horizon is inclined to the equator and its parallels at ſome angle leſs than 90°: in this caſe, the poſition of the ſphere is ſaid to be oblique.

If a tangent be drawn to the ſurfaces of the Sun and Earth, in the ſame plane with a line joining their centres, a circle deſcribed through the point of contact at the Earth's ſurface, the plane of it being perpendicular to the abovementioned line, will be the circle which terminates light and darkneſs.

The plane of this circle nearly, though not exactly, paſſes through the Earth's centre: it may be conſidered as a great circle without creating ſenſible error.

VII. When the Sun is in either of the equinoctial points, the circle bounding light and darkneſs coincides with the poles of the world, and conſequently biſects all the parallels to the equator.

Hence it follows, that when the Sun is in either of the equinoctial points, the days and nights in all parts of the world will be equal.

VIII. When the Sun's declination is north, the circle terminating light and darkneſs divides the northern parallels of latitude unequally, thoſe parts being the greateſt which lie within the ſaid circle.

Hence the days will be longer than the nights to the inhabitants of the northern parallels of latitude, when the Sun's declination is north.

IX. When the Sun's declination is ſouth, the circle terminating light and darkneſs divides the northern parallels of latitude unequally, that part of them being the greateſt, which lies without the circle of illumination.

For this reaſon the nights are longer than the days, to the inhabitants of the northern parallels, when the Sun is ſouth of the equator.

X. When the Sun's declination is greater than the complement of latitude to 90°, the parallel of latitude paſſing through the place, will fall either entirely within or entirely without the circle of illumination.

The Sun's centre will therefore appear to be entirely above or below the horizon to the inhabitants of that parallel, according as the declination is of the ſame, or of a contrary denomination to the latitude of the place.

CONCERNING THE MENSURATION OF ANGLES.

I. If two objects appear coincident with the extremities of a circular arc, to the eye of a ſpectator, ſituated at the centre of the circle, that arc will meaſure the obſerved angle ſubtended by the objects.

1. Let a repreſent any arc, A° the angle ſubtended by it, p a right angle, c the circumference, and r the radius of the circle; we have the following proportion:

$A° : 4p :: a : c$, wherefore $A° = \frac{4p \times a}{c}$; and ſince $4p$ is the ſame in all circles, A° will be proportional to $\frac{a}{c}$ or $\frac{a}{r}$; that is, the angle will be as the arc directly, and the radius of the circle inverſely.

2. Into whatever number of equal parts the circumference of a circle is divided, a right angle will be meaſured by $\frac{1}{4}$ of them; wherefore, if the circumference be divided into 360 equal parts or degrees, a right angle will be ſubtended by 90°.

3. The arcs which meaſure ſmall angles are nearly equal to their ſines, chords or tangents, the radius

of the circle being given: ſmall angles will therefore be nearly proportional to their ſines, &c. in the ſame circle, and in different circles will be as their ſines directly, and the radii of the circles inverſely.

4. Small angles ſubtended by a given rectilineal object, which is perpendicular to the axis of viſion, will be greater in the ſame proportion as the diſtances of the obſerver are leſs.

5. Let $B^{\circ} = 57^{\circ}.17'.45''$, or $20265''$ (being the angle which is ſubtended by an arc = radius).—Since it appeared by note 1, that $A^{\circ} = \frac{4p \times a}{c}$ and by the properties of the circle $\frac{4p}{c} = \frac{B^{\circ}}{r}$, it follows that $A^{\circ} = \frac{a}{r} \times B^{\circ}$. Hence $a = \frac{A^{\circ}}{B^{\circ}} \times r$.

By the preceding rules any two of theſe quantities, viz. a circular arc, the angle ſubtended by it, and the radius of the circle being known, the other may be determined.

The tables III. and IV. in the appendix will facilitate the eſtimation of arcs in terms of the radius and *vice verſa*: thus to find what angle is ſubtended by an arc $= \frac{7}{100}$ of an inch to a radius of 3 inches: here $A^{\circ} = \frac{B^{\circ}a}{r}$, and from table III. $.07 = 14438''.5$: dividing by 3 becauſe $r = 3$, we have the angle required $= 4812''.83 = 1^{\circ}.20'.12''.83$.

To find the length of an arc which ſubtends an angle of 7′ or 420″ to a radius = 3 inches. Here from table IV.

$$
\begin{array}{rcr}
400'' & = & .00193.925 \\
20 & = & 9.696 \\
\hline
420'' & = & .00203.621 \\
\hline
\end{array}
$$

this quantity being multiplied into the radius $r=3$ will give the length of $a=$ the arc required $=.00610.863$ parts of an inch.

Suppoſe it were required to aſſign the length of an arc $=1''$ on a great circle of the earth, the earth's radius being aſſumed equal to 3970 miles or 6987200 yards: here from table IV. we have $1''=00000.48481$ and which being multiplied into 6987200 will give the length required $=34$ yards nearly.

The arcs of quadrants and ſectors uſed for the menſuration of angles are divided into degrees, &c. and in inſtruments of a very large radius the diviſions are extended (by means of a nonius hereafter deſcribed) to ſeconds of a degree.—An index of equal curvature with the arc is applied to the extremity of a moveable radius, by means of which, the angle ſubtended by the objects is determined.

To the moveable radius abovementioned, a teleſcope is applied, having a fixed point marked in the centre of the field; a line joining this point and the centre of the object glaſs is called the line of collimation: this line determines the direction in which an obſervation is made.

II. An obſerved angle will err from the truth, if the line of collimation be inclined to the plane of the quadrant with which the obſervation is taken.

Suppoſe the quadrant to be fixed in any given plane; the line of the collimation may be conſidered as a radius of a ſphere, a great circle of which coincides with the arc of the quadrant.

If the line of collimation be inclined to the plane of the quadrant, it will, during the motion of the index, be continually directed to a leſſer circle of the ſphere, and therefore cannot determine the angular diſtance of objects, which is always meaſured upon a great circle.

It is ſuppoſed that every part of the graduated arc, and the index contiguous to it, lies in the ſame plane, which is called the plane of the quadrant, and is perpendicular to the axis of motion upon which the quadrant or the index applied to it, moves.

III. The plane of a quadrant being vertical, if one of the radii containing the right angle be horizontal, the other will be perpendicular to the horizon: an intermediate radius being directed to a ſtar, will divide the quadrant into any two arcs, one of which meaſures the zenith diſtance, and the other the ſtars altitude above the horizon.

Although the plane of the quadrant be vertical and coincident with that in which the line of collimation moves, yet the following errors may cauſe a difference between the true and obſerved altitudes.

1. If the line of collimation be not horizontal when the index points to 0 (or 90°).

2. If the diviſions of the quadrant are inaccurate.

3. If the diviſion pointed to by the index be read erroneouſly.

The direction of theſe errors coinciding with the plane of the quadrant ſhould be conſidered together. The firſt circumſtance which occurs is, that they do not neceſſarily combine to increaſe or diminiſh the obſerved altitude; but it may happen that their effects being contrary, ſhall correct, or even deſtroy each other.

In order to form an eſtimate of the error which will probably ariſe on this account, let us ſuppoſe, firſt that the diviſion of the quadrant is mathematically exact; and that the adjuſtment of the line of collimation to the point o, and the reading off can be depended upon to d'', but that the error may be of any magnitude leſs than d''.—It is an even probability which of the ſeconds between $+d'$ and $-d''$ is equal to the error on each account; the ſum of the two errors may ± be $2d''$, $\pm\,\overline{2d-1}''$, $\pm\,\overline{2d-2}''$, &c. of which combinations there are $4d^2$; among theſe there are $2d^2$, and no more, each of which exceeds $\frac{d''\times\sqrt{2}}{1+\sqrt{2}}$ (*d* being a very large number) from whence it is concluded that the probable error, or that which is an even chance the error of the obſervation does not exceed $= \frac{d'\times\sqrt{2}}{1+\sqrt{2}} = \frac{17d'}{29}$ nearly. If $d'' = 60''$ the probable error $= \frac{17\times 60''}{29} =$ about $35''$.

It appears likewiſe that if the error in the diviſion of the quadrant be taken into conſideration with the other two, and the limit of it be of equal magnitude with the former viz. d', the error of an obſervation compounded of the three errors above deſcribed, may be either $\pm\,3d''$, $\pm\,\overline{3d-1}''$, $\pm\,\overline{3d-2}''$, &c. of which combinations there are $8d^3$; among theſe there are $4d^3$ and no more, each of which exceeds $\frac{d'}{1.414}$ (*d* being a very large number) from whence it is inferred, that the probable error upon the whole $= \frac{d'}{1.414}$: If $d' = 60''$; the probable error $= 42''$ nearly.

IV. Although an aſtronomical quadrant be free from all the errors above deſcribed, if the plane of it ſhould be inclined to that of the vertical circle in which an obſerved ſtar is ſituated, the altitude deduced from obſervation will be greater than the true altitude.

Let the deviation of the quadrant's plane from that of the vertical circle be an angle (incomparably ſmaller than the altitude) the verſed ſine which $=u$, and let the ſine of the obſerved altitude $=s$, the coſine $=c$, radius $=1$, $M^{\circ}=57^{\circ}\cdot17'\cdot45''$: then the difference of the obſerved and true altitudes, ariſing from the inclination of the quadrant to the vertical circle, $=scu\times M^{\circ}$.

Cor. I. The deviation of the quadrant being given, the error $scu\times M^{\circ}$ will be the greateſt when the altitude $=45^{\circ}$, becauſe the product of the ſine and coſine of 45° is greater than the product of the ſine and coſine of any other angle.

Cor. II. If the altitude be the ſame, the error in obſerving it will be in a duplicate ratio of the deviation of the quadrant's plane from the vertical, becauſe the verſed ſines of arcs are in a duplicate ratio of the arcs themſelves, when they are diminiſhed *ſine limite*.

The obſerved altitude being a ſmall angle $=d'$, the condition of the propoſition is altered when the error of the quadrant bears a conſiderable proportion to d'. If u repreſents the verſed ſine of the quadrant's error as before, the error in altitude will now $=ud'$.

V. If the line of collimation be inclined to the plane of a quadrant otherwiſe accurately adjuſted, the altitude of a ſtar deduced from an obſervation taken by it will be greater than the true altitude.

If the inclination of the line of collimation to the plane of the quadrant be an angle (incomparably ſmaller than the zenith diſtance) the verſed ſine of which $=u$,

$=u$, let T = the tangent of the obſerved altitude, $M^{\circ}=57^{\circ}.17'.45''$; the difference between the true and obſerved altitudes $=uT\times M^{\circ}$.

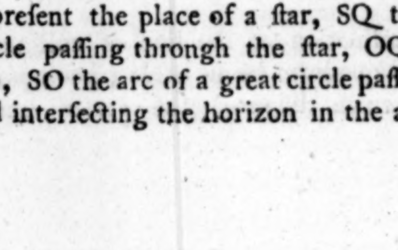
Cor. I. The altitude being given, the error in obſerving it will be in a duplicate ratio of the error of the line of collimation.

Cor. II. The error of the line of collimation being given, the difference between the obſerved and true altitudes will be as the tangent of the altitude.

When the obſerved zenith diſtance is ſo ſmall that the error of the line of collimation bears a conſiderable proportion to it, let $e=$ the error of the line of collimation, $d=$ the ſmall zenith diſtance, deduced from obſervation: the error in altitude now becomes $=\sqrt{d^2+e^2}-d$.

VI. Suppoſe a great circle to paſs through a ſtar, and to interſect the horizon at any angle; if this angle and the arc of the great circle intercepted between the ſtar and the horizon be taken by obſervation, the ſtar's altitude above the horizon will be determined.

This indirect method of meaſuring the altitudes of objects is attended with conſiderable advantages.



Let S repreſent the place of a ſtar, SQ the arc of a vertical circle paſſing through the ſtar, OQ an arc of the horizon, SO the arc of a great circle paſſing through the ſtar and interſecting the horizon in the angle SOQ:

according to the method deſcribed in the article, the arc OS and the angle SOQ are obſerved in order to determine the altitude SQ: It appears from eſtimating the variations in the parts of the triangle SOQ that an error $= d''$ in meaſuring the arc OS will produce an error $= \frac{d' \times \text{tang. SQ}}{\text{tang. OS}}$, in the arc SQ deduced from the values of SO and SOQ. It appears alſo that an error $= d''$ in the angle SOQ will produce an error $= \frac{d' \times \text{tang. SQ}}{\text{tang. O}}$; conſequently the ſum of theſe errors in the vertical arc $\text{SQ} = \frac{d' \times \text{tang. SQ}}{\text{tang. OS}} + \frac{d' \times \text{tang. SQ}}{\text{tang. O}}$.

It is plain that to meaſure a given altitude SQ any different values of SOQ or OS may be aſſumed; it becomes therefore a ſubject of enquiry, having given the arc SQ, what muſt be the value of SOQ or SO, ſo that the ſum of the errors which affect SQ may be the leaſt poſſible. The inveſtigation need not be here inſerted, but the reſult is this. d' being a conſtant quantity $\frac{d' \times \text{tang. SQ}}{\text{tang. OS}} + \frac{d' \times \text{tang. SQ}}{\text{tang. O}} = \dot{\overline{\text{SQ}}}$ is a minimum when the ſine of the angle SOQ is a mean proportional between the ſine of SQ and radius; provided the arc SQ is leſs than 30°: but when SQ exceeds 30°, the error juſt mentioned becomes a maximum. Let the ſine of $\text{SQ} = s$, radius $= r$. Then whatever be the value of SQ the quantity $\dot{\overline{\text{SQ}}} = 2d'' \times \sqrt{\frac{s}{r+s}}$ on a ſuppoſition that the two variations of SOQ and SO both combine in their effects to increaſe or to diminiſh the value of SQ: but ſince they may with equal probability tend to correct or even deſtroy each other, from the principle contained in Art. III. it may be collected that it is an even chance that the error does not exceed $\frac{d'' \times \sqrt{2}}{1+\sqrt{2}} \times \sqrt{\frac{s}{r+s}} = \frac{17d''}{29} \times \sqrt{\frac{s}{r+s}}$ nearly; whereas the probable error ariſing in the direct menſuration of the vertical arc SQ $= \frac{d''}{2}$ which is always greater than $\frac{17d''}{29} \times \sqrt{\frac{s}{r+s}}$ whatever be the value of SQ: but in determining the ſmaller altitudes by the indirect method, the probable error will be reduced within very narrow limits; for example, in meaſuring an altitude of 1°; if the obſervations of OS and SOQ are liable to an error which may ariſe to but not to exceed 60″ the probable error in the altitude deduced will be $= \frac{17 \times 60''}{29} \times \sqrt{\frac{\text{ſine } 1''}{r \times \text{ſine } 1^\circ}} = 4''.6$, whereas the probable error in meaſuring the ſame altitude directly would be 30″.

The equatorial inſtrument which has an azimuth circle may be very eaſily applied to this method of obſervation.

VII. If any line conſiſts of equal parts, each of which is $= p$, and it is required to ſubdivide each of the diviſions p into n equal parts, let a nonius $= \overline{n+1}$, or $\overline{n-1}$ parts each $= p$ be aſſumed, and divide the nonius into n equal parts: applying it to the ſpace $\overline{n \pm 1} \times p$ ſo that the extremities may coincide, it will follow that the diſtance between the firſt two correſponding diviſions from coincidence will be $\frac{p}{n}$; the diſtance between the two next, $\frac{2p}{n}$, the diſtance between the two next, $\frac{3p}{n}$, and ſo on till the diſtance between the two laſt $= \frac{np}{n} = p$.

Suppoſe the arc of a quadrant were ſuch as could be divided conveniently into arcs each equal to the $\frac{1}{8}$ part of a degree or 7′30″: to conſtruct a nonius which ſhall expreſs arcs to every 30″; here becauſe 30″ is contained 15 times 7′ 30″, it will follow that $n=15$, wherefore the length of the nonius muſt be either $16 \times 7' 30'' = 2^\circ$, or $14 \times 7' 30'' = 1^\circ 45'$; either of theſe being aſſumed for the length of the nonius, if it be divided into 15 equal parts, and its extremities are applied contiguous to any two correſponding diviſions on the arc, the diſtance between the two firſt diviſions from coincidence will be 30″ between the two next 60″ &c.

The nonius applied to aſtronomical and other inſtruments is conſtructed upon theſe principles.

By means of this contrivance, angles are obſerved to an accuracy, much beyond what could be effected by actual diviſion of the quadrant.

If ſeveral obſervations be taken in order to determine the angle ſubtended between two objects, the reſults of theſe obſervations will probably differ ſomething from each other: It is uſual in this caſe to add the reſults together, and to divide the ſum by the number of them: the quotient is called the mean of the whole, and every

thing

thing elſe being the ſame, will approach nearer to the truth, the greater number of obſervations are taken, from which the mean is derived. Thus, if p, q, and r be the reſults of three obſervations, the mean of the three will be $\frac{p+q+r}{3}$.

VIII. The angle ſubtended by two objects obſerved with a Hadley's quadrant is double to the angle at which the reflecting planes are inclined to each other at the time of obſervation.

The line of collimation ſhould be parallel, and the reflecting planes perpendicular to the plane of motion; if they deviate from theſe poſitions, by a ſmall angle, the errors in the obſerved angle may be eſtimated thus:

Let v be the verſed ſine of the angle, at which the line of collimation is inclined to the plane of motion: let p be the ſine of $\frac{1}{4}$ part of the obſerved angle; the error of the obſerved angle will be ſubtended by an arc $= \frac{4pv \times \sqrt{1-p^2}}{1-2p^2}$ to radius $= 1$.

To eſtimate the error ariſing from the deviation of the reflecting planes from the perpendicular to the plane of motion, let the verſed ſine of that deviation be v, let $m =$ the ſine of the angle of incidence at the fixed ſpeculum; then p being the ſine of $\frac{1}{4}$ of the angle meaſured, the error of the obſerved angle will be meaſured by an arc $=$

$$\frac{4pv \times \overline{2 - 3p^2 - m^2 + 2p^2m^2 \mp 2mp \times \sqrt{1-m^2} \times \sqrt{1-p^2}}}{\sqrt{1-p^2} \times \overline{1-2p^2}}$$

IX. The apparent altitudes of all the heavenly bodies are increaſed by refraction, except when they are ſituated in the zenith.

An obſerved altitude of any ſtar or other object in the heavens muſt be diminiſhed by a ſmall angle taken from a table of refractions calculated for the different altitudes above the horizon. A table of this kind is inſerted in the Appendix, N° V.

The obſervation of a ſtar's altitude implies the previous determination of the horizon. This is effected two ways, 1. by a plumb line hanging freely and at reſt; a plane perpendicular to this line will be horizontal. 2. By the level.

If a ſtar is obſerved by reflexion from an horizontal poliſhed ſurface, half the angle ſubtended by the image and object will be equal to the ſtar's altitude above the horizon.

To find the altitude of the Sun's centre, two methods are uſed: 1. By obſerving the altitudes of the higheſt and loweſt points of the diſk, half the ſum will be the altitude of the Sun's centre. 2. By obſerving either the higheſt or loweſt point only, and ſubtracting or adding the Sun's ſemi-diameter correſponding to the time of obſervation taken from an ephemeris in which it is previouſly calculated.

ARTICLES RELATIVE TO THE PRACTICE OF ASTRONOMY.

I. If equal altitudes of a fixed ſtar are taken during its apparent diurnal revolution, the time elapſed between the obſervations being biſected will give the inſtant of the ſtar's meridian tranſit.

To obſerve ſtars when they are on the meridian, an inſtrument is made uſe of called a tranſit teleſcope, which is ſo adjuſted, that, while it turns upon a fixed axis, the line of collimation ſhall deſcribe the plane of the meridian: to effect this, the line of collimation muſt be firſt adjuſted perpendicular to the axis upon which it turns: the axis muſt then be placed horizontal, ſo that the line of collimation, being turned round it, may deſcribe a vertical circle; the axis continuing horizontal, the inſtrument is then to be adjuſted to the plane of the meridian in the manner deſcribed in the article. If the inſtant of the ſtar's tranſit does not biſect the time elapſed between the obſervations, the poſition of the tranſit muſt be corrected.

The meridian may alſo be determined by taking the azimuths of any fixed ſtar when the altitudes are equal; the difference of the azimuths being biſected will give the place of the meridian.

II. The time between two ſucceſſive tranſits of the ſame fixed ſtar over the ſame meridian is called a ſidereal day.

Any equable motion may be aſſumed as a meaſure of time: but, ſtrictly ſpeaking, no ſuch motion is known, except that of the angular revolution of the earth round its axis. Motions called equable, in a practical ſenſe, which are conſtructed for the meaſurement of time, are liable to imperfections and irregularities, and are all either immediately or remotely corrected by the angular motion of the Earth round its axis, or, which is the ſame, by the apparent diurnal revolutions of the fixed ſtars.

The ſidereal day may be divided into any number of equal parts; it is uſually divided into 24 hours, each hour into 60 minutes, and each minute into 60 ſeconds; conſequently, a fixed ſtar will deſcribe an arc referred to the equator equal to 15° in one of the ſidereal hours, 15′ in a minute, &c.

III. If the time elapſed between the tranſits of two ſtars over the meridian be obſerved,

ſerved, the angular difference of their right aſcenſions will be obtained from the following proportion : as the time of a ſidereal day is to the time elapſed between the tranſits, ſo is 360° to the angular difference of right aſcenſion.

By obſerving the Sun's tranſits over the meridian his motion in right aſcenſion is obtained.

IV. Having given the time of a ſtar's tranſit over the meridian, and the times of its paſſing over two vertical circles at given (azimuth) diſtances from the meridian, to determine the latitude.

The difference of the times between the ſtar's tranſit over the meridian and over the other verticals, will give the angles contained between the meridian and the circle of declination paſſing through the ſtar, by this proportion : as a ſidereal day is to the time elapſed between the tranſit over the meridian and the vertical circle, ſo is 360° to the angle contained between the meridian and circle of declination paſſing through the ſtar, or, as it may be termed, the angle at the pole.

Let the ſmalleſt angle at the pole be E, correſponding to the firſt obſervation ; the ſtar's azimuth obſerved at the ſame time $=$P: let the larger angle at the pole be F, the azimuth of the ſtar correſponding $=$ Q. Then the ſine of the latitude of the place $=$

$$\frac{\text{Cotang. } Q \times \text{ ſine } F - \text{cotang. } P \times \text{ ſine } E}{\text{Coſine } E - \text{coſine } F}.$$

Thus, ſuppoſe a ſtar was obſerved to paſs two vertical circles at 34° and 43° azimuth diſtances from the meridian, the times of obſervation from the meridian tranſit of the ſtar being $1^h\ 32^m$, and $3^h\ 32^m$, eſtimated by ſidereal time reſpectively : by converting the given times into degrees we have, from the preceding expreſſion, $E=23°$, $F=53°$, $P=34°$, $Q=43°$, and the ſine of the latitude required,

$$=\frac{\text{Cotang. } 43° \times \text{ſine } 53° - \text{cotang. } 34° \times \text{ſine } 23°}{\text{Coſine } 23° - \text{coſine } 53°}$$

$$=\frac{.8564316-.5792828}{.9205049-.6018150}= \text{ſine of } 60°\ 25'\ 5''$$, the latitude required.

Other methods of finding the latitude of a place are more eligible and eaſy than that which has juſt been deſcribed: but they require the previous knowledge of the effects produced by refraction, in elevating objects above their true places, of which the determination of the latitude from the obſervation of two azimuths, and the times correſponding is wholly independent.

V. The meridian and latitude of a place being known, the refraction in altitude of a ſtar when at any given diſtance from the horizon may be determined.

Let the ſtar's apparent altitude and azimuth with the time from the meridian correſponding be obſerved: The complement of latitude, the angle at the pole, and the azimuth, are three parts of a ſpherical triangle, from which the third ſide may be found; the ſolution of the triangle will give the true zenith diſtance, and true altitude of the ſtar at the time of obſervation, which being ſubtracted from the apparent altitude, the difference will be the refraction in altitude.

Having given tables of refraction, the apparent altitudes of objects muſt be corrected to obtain the true altitudes.

VI. The latitude of a place, is equal to the altitude of the pole of the equator above the horizon.

1. The diſtance between the zenith and the horizon is equal to the diſtance between the pole and the equator, each of them being 90°: take away the diſtance of the zenith from the pole, which is common to both, and there remains the diſtance of the pole from the horizon, equal to the diſtance of the zenith from the equator, or the latitude of the place.

2. The diſtance of the zenith from the pole is equal to the complement of the latitude to 90°.

3. It appears by experiment, that an obſerver, who proceeds directly on the meridian 69.3 miles in the latitude of 57°, changes his latitude 1°. From whence it is inferred, ſuppoſing the Earth to be perfectly ſpherical, that an arc of one degree on the Earth's ſurface $=69.3$ miles, and conſequently that the Earth's circumference $=69.3 \times 360 = 24948$, and the diameter $=7941$ miles.

The ſpheroidal figure of the Earth cauſes a ſmall difference in the length of a degree, when meaſured on a meridian circle of the Earth in different latitudes.

VII. If the greateſt and leaſt altitudes of a circumpolar ſtar be determined by obſervation, half the ſum will be the latitude of the place.

The ſtar uſually called the pole ſtar, is not exactly in the pole of the equator; the above method is therefore (among others) made uſe of to determine the true altitude of the pole.

VIII. The declination of a ſtar is the difference between (or ſum of) the ſtar's meridian zenith diſtance and the latitude of the place.

The latitude of a place being known, the declinations of the ſtars are determined from the obſervation of their meridian zenith diſtances.

IX. The inclination of the equator to the horizon, is equal to the complement of the latitude to 90°.

The diſtance of the zenith from the horizon = 90°. The meridian diſtance of the zenith from the equator = the latitude: the arc of the meridian therefore intercepted between the equator and the horizon muſt be = the complement of the latitude to 90°. This arc meaſures the inclination of the equator to the horizon, becauſe the meridian is a ſecondary common to both thoſe circles.

X. The latitude of a place is equal to a ſtar's meridian zenith diſtance added to the declination, if the ſtar paſſes the meridian between the zenith and the equator: and in all other caſes the latitude is the difference between the meridian zenith diſtance and declination of the ſtar.

In this article the ſtar is always ſuppoſed to paſs the meridian above the pole.

The following is an example of the proceſs for finding the latitude from an obſervation of the Sun's altitude taken at Cambridge. Aug. 7, 1776.

	°	′	″
Apparent meridian altitude of the Sun's lower limb,	53	46	8
Sun's apparent ſemi-diameter from Ephemeris,		15	50
Apparent altitude of the Sun's centre,	54	1	58
Subtract on account of refraction,			41
Altitude of the Sun's centre =	54	1	17
Zenith diſtance of the Sun's centre,	35	58	43
Add the Sun's declination,	16	13	57
Latitude of Trinity College obſervatory	52	12	40

When great exactneſs is required, the effects of the air's weight and temperature, and the Sun's parallax, muſt be taken into account.

Altitudes taken at ſea require a correction, the line joining the obſerver's eye and the apparent horizon being below the level of the horizontal plane.

XI. The altitude of an object taken at ſea is greater than the true altitude, and the difference is found by this proportion: as the ſum of the Earth's ſemi-diameter and the elevation of the eye above the level of the ſea, is to the Earth's ſemi-diameter, ſo is radius to the coſine of the difference required.

If r = the Earth's radius, and a = the altitude of the obſerver's eye above the level of the ſea, the coſine of the dip will be $= \frac{r}{r+a}$ to radius $= 1$.

Thus ſuppoſe the altitude of the obſerver above the ſea to be 30 feet $= a$: becauſe the Earth's radius is about 3970 miles $= r$, the coſine of the dip will be $= \frac{3970 \times 5280}{3970 \times 5280 + 30}$ = coſine of $5' \, 49''$: which angle muſt be ſubtracted from altitudes taken at ſea, when the obſerver is elevated 30 feet above the ſea's ſurface. If the elevation is decreaſed, the angle of the dip will be diminiſhed in a ſubduplicate proportion without ſenſible error. Thus, if the obſervation is taken from an elevation of 15 feet, the dip will be only $\frac{5' \, 49''}{\sqrt{2}} = 4' \, 7''$. See Table VI. in the Appendix.

XII. The inclination of the ecliptic to the equator, is equal to the angle which is meaſured by the Sun's greateſt declination.

The Sun's declination increaſes ſo ſlowly when he is near the ſolſtices, that the declination deduced from a meridian altitude taken within a few hours of the ſolſtice will not differ much from the greateſt declination. The interſection of the ecliptic with the equator correſponding to the firſt point of Aries, is the point from which the longitudes and right aſcenſions of the Sun and ſtars are eſtimated.

The time of obſervation in the preceding articles may be meaſured by any equable motion without immediate reference to the heavenly bodies; but, in general, time is referred to the Sun's motion, becauſe the diviſion of the day and night and the difference of ſeaſons depend on that luminary; except for theſe reaſons, it is not probable that the Sun's motion would have been adopted as a meaſure of time, being diſturbed by various irregularities, for which it is neceſſary to apply corrections in order to make it correſpond with an uniform motion.

XIII. The time elapſed between the centre of the Sun's paſſing any meridian, and returning to the ſame meridian, is called an apparent or ſolar day.

When the Sun's centre paſſes the meridian, it is called noon day; the Sun continues to riſe in the heavens until noon, when he has attained the greateſt altitude above the horizon; he afterwards deſcends in his motion toward the weſt.

The time of noon is inferred from obſerving the times when the Sun's eaſtern and weſtern limbs are on the meridian; this time being biſected will give the inſtant of apparent noon.

XIV. The ſiderial day is obſerved to be leſs than the ſolar day.

If the Sun were ſtationary among the fixed ſtars, the ſolar and ſiderial days would be equal: But during the time elapſed between any meridian of the Earth having paſſed, and again returning to the Sun's centre, the Sun has changed his place in the heavens, by moving toward the eaſt; on which account the Earth, beſides one entire revolution of 360°, muſt have deſcribed an additional angle equal to that which the Sun has moved through eſtimated in right aſceuſion.

XV. The proportion between a ſolar day and a ſiderial day will be known, if the increaſe of the Sun's right aſcenſion during the ſolar day is given.

Suppoſe this increaſe of right aſcenſion to be d; let the time of a ſiderial day be t; the following proportion will give the time of the ſolar day required: viz.

As $360° : 360°+d :: t$ to the ſolar day $= t \times \frac{360°+d}{360°}$.

If therefore d, or the increaſe of the Sun's right aſcenſion during the time elapſed between his paſſing and returning to a given meridian, were always the ſame, the ſolar days would be all equal, t and 360 being conſtant quantities; but the Sun's motion in right aſcenſion being unequal, it is plain that the ſolar days muſt be of unequal lengths, depending on the values of the quantity d. To reduce the ſolar time to mean time, it is requiſite, that the length of a mean ſolar day and a mean value of the Sun's increaſe in right aſcenſion during that mean ſolar day ſhould be determined. Unequal quantities are reduced to a medium, by adding a great many together and dividing the ſum by the number of quantities: thus a mean ſolar day is obtained, by aſſuming

the

the time elapſed during any large number; for example, 36525 unequal apparent ſolar days; this time divided into 36525 equal parts will be the length of a mean ſolar day. In the ſame manner, the motion of the Sun during a mean ſolar day, will be obtained by obſerving the Sun's motion in longitude or right aſcenſion, during a great number of days, for example, 36525, and dividing the Sun's motion ſo obſerved into 36525 equal parts.

The ſmall difference between the Sun's mean motion in longitude and right aſcenſion cauſed by the preceſſion of the equinoxes not here conſidered.

XVI. The Sun's motion in longitude during one day, is at a medium = 59′ 8″.330.

The Sun's motion during 100 Julian years or 36525 days has been obſerved to be 100 revolutions and 46′ 10″ or 36000° 46′ 10″; which being divided by 36525, the quotient will be the Sun's mean motion in longitude or right aſcenſion during one mean ſolar day, which will be found = 59′ 8″.330 as is expreſſed in the article.

Let the length of a mean ſolar day be m, and let t be the length of a ſiderial day; the properties of equable motion give this proportion:

As 360° : 360° 59′ 8″.33, ſo is t to m, wherefore

$$m = t \times \frac{360^\circ\ 59'\ 8''.33}{360^\circ}, \text{ or } t = m \times \frac{360^\circ}{360^\circ\ 59'\ 8''.33}$$

the proportion between the ſiderial and mean ſolar days being determined, either may be expreſſed in dimenſions of the other.

If the mean ſolar day is divided equally into 24 hours, the ſiderial day will be $= \frac{24^h \times 360^\circ}{360^\circ\ 59'\ 8''.33} =$ $23^h\ 56^m\ 4^s.09$; and if this quantity is ſubtracted from 24^h, the difference will be $3^m\ 55^s.91$ being the time by which any given fixed ſtar paſſes the meridian ſooner on each day than on the day preceding.

XVII.

XVII. If the increaſe of the Sun's right aſcenſion during any apparent ſolar day with the increaſe of the Sun's right aſcenſion during one equated or mean ſolar dty of 24^h be given, the length of the apparent ſolar day will be determined.

Let d be the increaſe of right aſcenſion given, $a =$ the Sun's mean increaſe of right aſcenſion during 24 hours: then the length of the ſolar apparent day will be determined by this proportion:

As $360 + a : 360 + d :: 24^h :$ to the length of the ſolar apparent day; which will therefore $=$

$$24^h \times \frac{360^\circ + d}{360^\circ + a}.$$

When d and a are equal, it is evident that the true or equated, and ſolar apparent days are equal, in which caſe, ſolar apparent time will be the ſame with true or equated time during any parts of thoſe days. On account of the variation in the quantity d, the ſolar apparent day will be greater or leſs than the equated day of 24^h according as d is greater or leſs than a, that is, according as d is greater or leſs than $59' \; 8''.33$.

XVIII. The inequality of the ſolar days renders a correction or equation neceſſary to reduce them to the ſame length.

If the Sun's motion in right aſcenſion were always equal to his mean or equable motion in longitude, which is at the rate of $59' \; 8''.33$ for each day, no correction would be required: conſequently the correction neceſſary muſt be the difference between the Sun's true right aſcenſion and mean longitude; a ſmall allowance being made on account of the preceſſion of the equinoxes, reduced to motion in right aſcenſion according to the method explained by the Rev. Dr. Maſkelyne, A. R. in the Philoſophical Tranſactions, Vol. LIV. p. 342.

XIX. From having obſerved the tranſit of the Sun's eaſtern and weſtern limbs over the meridian, it is required to aſſign the time of apparent noon and the true time of noon day.

Add together the times correſponding to the two obſervations as ſhewn by a clock, and take half the ſum, this will give the time of apparent noon: if the equation be applied with its proper ſign to the time of apparent noon, the reſult will be the true time of noon day. If at the inſtant of the true noon the clock points to XII. it wants no correction; if otherwiſe, the error of the clock is diſcovered.

XX. Having given the angle contained between the meridian and a circle of declination paſſing through the Sun's centre, to aſſign the apparent and true time of day.

The given angle converted into time, at the rate of 15° to 1^h, $15'$ to 1^m, &c. will give the apparent time of day: let this equal A, and to obtain the true time, let ± E be the equation of time for preceding noon; $\pm e =$ the variation of the equation during the next 24^h; the true time will be $= A \pm E \pm \frac{eA}{24^h}$.

To facilitate the computation of the quantity $\frac{e \times A}{24^h}$, a table is inſerted in the Appendix, N° VII. containing hours, minutes, and ſeconds, expreſſed in parts of a day. Having given the time expreſſed in decimals of a day, and the equation for preceding noon with the variation of the equation during 24^h, the equation correſponding to the given time of day will be obtained from one multiplication only. Thus ſuppoſe the apparent ſent given time to be $9^h\ 53^m$, this is reduced to decimals of a day by referring to the table:

9^h	.375
50^m	.035
3^m	.002
	.412

Let the equation for preceding noon be $4^m\ 3^s.1$, and its increaſe during the ſucceeding $24^h = 8^s.2$; by multiplying .412 into 8.2 the product will be $3^s.38$, which being added to $4^m\ 3^s.1$ the ſum will $= 4^m\ 6^s.5 =$ the equation correſponding to the given time. (N. B. The proportional parts for the RA and equation may be calculated from the ſame proceſs by Table IX. and VII.) This table is uſed exactly in the ſame manner for finding the declination or right aſcenſion correſponding to any given time, the declination or right aſcenſion at noon being known with the variation during 24^h.

The angle contained between the meridian and a circle of declination paſſing through the Sun's centre is obtained from an obſervation of the Sun's altitude; this will give the Sun's zenith diſtance, which arc with the complement of latitude and the complement of declination will form a ſpherical triangle, the three ſides being given, from which either of the angles may be obtained. The uſual proceſs for finding the hour angle is ſubjoined in the following example:

Suppoſe the Sun's zenith diſtance to be by obſervation $= 76°3'$, the complement of the latitude to $90° = 37°47'$ and the complement of the Sun's declination to $90° = 66°\ 58'$.

		°	′		
Comp. lat.	=	37	47	log. coſecant	= 0.2127683
Comp. dec.	=	66	58	log. coſecant	= 0.0360813
Zen. diſt.	=	76	3	l. ſine of ½ ſum	= 9.9999894
Sum	=	180	48	log. ſine diff.	= 9.3941794
½ Sum	=	90	24	Sum of 4 logarithms.	= 9.6430185
Subtr. zen. diſt.	=	76	3		
				½ Sum	= 9.8215092
Difference	=	14	21		

= the log. coſine of 48° 28′ 17″ the double of which is 96° 56′ 34″ = the hour angle required.

Or the following method may be uſed: let the difference between the latitude and Sun's declination be A; and find the logarithm of $\overline{\text{Cos. A} - \text{ſine of the Sun's altitude}}$, this added to the logarithmic ſecants of the latitude and declination will be the log. verſed ſine of the hour angle.

Referring to the preceding example.

$$
\begin{array}{lr}
\text{A} = 29^\circ 11' \text{ and log. } \overline{\text{cos. } 29^\circ 11' - \text{ſin. } 13^\circ 57'} = & 9.8007094 \\
\qquad\quad \text{Log. ſecant } 52^\circ 13' \qquad\quad = & 0.2127683 \\
\qquad\quad \text{Log. ſecant } 23 \quad 2 \qquad\quad = & 0.0360813 \\
\hline
 & .0495590
\end{array}
$$

= the verſed ſine of 96° 56′ 34″ correſponding to $6^h\ 27^m\ 46^s.2$ the apparent time of day when the obſervation was taken.

Theſe are the methods of finding apparent time uſually practiſed at ſea—the problem may alſo be effected by obſerving the altitude of a kuown fixed ſtar with very little addition to the computation (infra). In each caſe, ſince it is impoſſible to meaſure altitudes above the horizon abſolutely free from error, the obſervation muſt be taken in ſuch a manner, that a given variation in the time deduced: which is done by taking the altitude of the Sun or ſtar when on the prime vertical. Let a = the given variation of a ſtar's altitude, b = the correſponding variation of the hour angle, then we ſhall have

$$b = a \times \frac{\overline{\text{radius}}^{2}}{\text{Cos. latitude} \times \text{ſine azimuth}}.$$

Therefore ſince at a given place $\frac{\overline{\text{rad.}}^{2}}{\text{cos. lat.}}$ is conſtant, and a being by the ſuppoſition invariable; h, or the variation of the hour angle deduced from obſervation will be proportional to $\frac{1}{\text{ſine azimuth}}$, which quantity is the leaſt when the azimuth $= 90^\circ$, or when the Sun or ſtar is ſituated on the prime vertical.

Cor. I. It appears from hence that (every thing elſe being the ſame) it is of no conſequence to the accuracy of the time deduced, at what altitude a ſtar is obſerved upon the prime vertical.

Cor. II. In the latitude of $52^\circ\ 12'\ 40''$, an error of $1'$ in obſerving the altitude of a ſtar upon the prime vertical cauſes an error of $\frac{1'}{\text{coſ. } 52^\circ\ 12'\ 40''} = 97''.9 =$ ſix ſeconds and a half in the time deduced.

If the obſervation be taken on any other vertical, the error in altitude being the ſame, the error in time before found ($6\frac{1}{2}$ ſeconds) will be increaſed in the ratio of the ſine of the ſtars azimuth to radius.

XXI. If the difference of the apparent time of day at any two places be known (at the ſame inſtant) their difference of longitude will be known.

One of the methods uſed for determining the difference of longitude, is by two perſons obſerving an eclipſe of the Sun, one of Jupiter's ſatellites, &c. or the occultation of a fixed ſtar by the Moon. If either of theſe phenomena happened at different apparent times to the two obſervers, the difference of theſe times (allowing for the effects of parallax, if neceſſary) will give the difference of longitude required. This method can be practiſed only on land. The following methods are uſed at ſea: a watch well regulated to time at a given place, being carried to ſea eaſtward or weſtward, will ſhew the apparent time (by means of adding or ſubtracting the equation) different from that which is deduced from obſervation of the Sun or ſtars. This difference of time converted into degrees and minutes is the difference of longitude required. The difference of longitude is now uſually determined at ſea by obſerving the angle ſubtended by the Moon's centre and a fixed ſtar, not very diſtant from the Moon's orbit.

The Moon's diſtances from ſeveral of the fixed ſtars, and the times correſponding, are calculated in the Nautical Ephemeris for the meridian of Greenwich; by obſerving the diſtance of the Moon from a fixed ſtar, the time at Greenwich may be determined; and the time at the place of obſervation being known, the difference converted into degrees and minutes, will be the longitude from Greenwich.

In this caſe it is neceſſary to obtain the true diſtance of the Moon from the ſtar, free from the effects of parallax and refraction, which alter the apparent altitudes of the ſtar and Moon, and cauſe a variation in their apparent diſtances. The apparent altitudes of the Moon's centre and a ſtar, with their apparent diſtances from each other, being given by obſervation, we obtain the true difference of their azimuths, which is not affected by refraction or parallax. The apparent zenith diſtances of the Moon and ſtar being known, their elevations ariſing from refraction, as well as the depreſſion of the Moon's centre cauſed by parallax, will be known from the tables, from whence we derive their true zenith diſtances: and the difference of the azimuths being previouſly determined, the ſolution of a ſpherical triangle gives the true diſtance of the Moon's centre from the ſtar.

XXII. Having the tranſit of a fixed ſtar over the meridian, to find the true time of day.

Putting RA as an abbreviation for right aſcenſion expreſſed in time, ſubtract the Sun's RA at preceding noon from the ſtar's RA increaſed by 24^h if leſs than the Sun's RA: let the difference be D: find the proportional parts for D from Table VIII. in the appendix and let them be repreſented by pD: if $\pm e$ be the equation of time for preceding noon, the true time of the ſtar's meridian tranſit will be $D-pD\pm e$. Vid. a paper by A. Aubert, Eſq. inſerted in the Philoſophical Tranſactions for 1776.

Suppoſe the ſtar's RA in time to be $5^h\ 0^m\ 37^s$: the Sun's RA at preceding noon $= 10^h\ 6^m\ 8^s.7$, and the equation of time for preceding noon $= +\ 2^m\ 31^s.3$: the proceſs for finding the time of the ſtar's meridian tranſit will be as follows:

	h		h	m	s	
From	24	+	5	0	37	
Subtract			10	6	8.7	
			18	54	28.3	= D
p D from Table VIII.				−3	5.9	
			18	51	22.4	
e =				+ 2	31.3	
			18	53	53.7	

Proportional parts for D, from Table VIII.

10^h	1^m	$38^s.3$
8	1	18.6
50^m		8.2
4		.7
30^s		.1
pD =	3	5.9

XXIII. Having given the diſtance D of a fixed ſtar from the meridian (reckoning on the equator eaſtward) to find the time.

Let the ſtars RA – Sun's RA at noon preceding –D=A: let *p*A be the proportional parts for A from Table VIII. and let ±*e* be the equation of time for noon preceding. Then the time ſought will be A–*p*A±*e*.

Or thus: let D be the diſtance of the ſtar from the meridian reckoning weſtward; find the time of the ſtar's preceding meridian tranſit, and let it = T: find *p* D from Table VIII. and the time ſought will be T+D–*p*D.

Let the given diſtance from the meridian be $3^h\ 40^m$ weſt, or $20^h\ 20^m$ reckoning eaſtward: let the ſtar's RA = $5^h\ 0^m\ 37^s$; ſuppoſe the Sun's RA at preceding noon to be $10^h\ 6^m\ 8^s.7$, and the equation of time = $2^m\ 31^s.3$.

	h	m	
To the given diſtance	20	20	
Add the Sun's RA	10	6	8.7
	6	26	8.7
Subtract this ſum from the ſtar's RA + 24^h	29	0	37
	22	34	28.3 = A
*p*A from Table VIII.		3	42
	22	30	46.3
Equation		+ 2	31.3
True time ſought =	22^h	33^m	$17^s.6$

	h	m	s	
Or thus: The time of the ſtar's preceding meridian tranſit is found from the laſt article =	18	53	53.7	= T
Add the given diſtance = D reckoning weſtward	3	40		
	22	33	53.7	
p D from Table VIII.			–36.1	
True time =	22	33	17.6	

XXIV. Having given the true time of day = T, to find the diſtance of a ſtar from the meridian reckoning on the equator weſtward.

Find the time of the ſtar's preceding meridian tranſit and let it be repreſented by *t*: let $T - t = A$, and from Table IX. take pA; the diſtance ſought, eſtimated in time, will be $A + pA$.

	h	m	s
Suppoſe the given time to be	22	33	17.6 = T
The time of the ſtar's preceding tranſit	18	53	53.7 = *t*
A =	3	39	23.9
*p*A from Table IX. =			+36.1
The diſtance required	3	40	00

XXV. To determine the angular apparent motion of a circle of declination paſſing through a fixed ſtar in its diurnal revolution during any given time T.

From Table IX. find pT and $T + pT$ converted into degrees and minutes will be the angular motion of the ſtar in the time T.

Thus if the difference of the true times at which two fixed ſtars paſs the meridian, is obſerved to be $9^h\,0^m\,35.^s3$, to infer the difference of their right aſcenſions.

	h	m	s
T =	9	0	35.3
From Table IX. *p*T =		1	28.8
The difference of right aſcenſions required	= 9^h	2^m	$4^s.1$

XXVI. Having given any arc of the equator D, to find the time in which a circle of declination, paſſing through a given fixed ſtar, deſcribes the arc D on the equator.

D being expreſſed in time find pD from Table VIII. and $D-pD$ will be the time required.

Thus, let the difference in right aſcenſion of two fixed ſtars be $9^h\ 2^m\ 4^s.1$: to find the time elapſed between their tranſits over a given meridian.

	h	m	s
Here D =	9	2	4.1
pD from Table VIII. =		−1	28.8
	9	0	35.3

Wherefore the time required between their meridian tranſits $= 9^h\ 0^m\ 35.^s3$.

ON DIALLING, AND THE USE OF THE EQUATORIAL INSTRUMENT.

I. If rays of light, proceeding from the Sun's centre, ſhine upon a right line parallel to the Earth's axis, the ſhadow of the line will be in the ſame plane with the circle of declination which paſſes through the Sun's centre.

The ſhadow of any line is a plane which paſſes through the line, and through the Sun's centre: the apparent time of day being the ſame, the ſhadow deſcribed in the article will cut the horizon, or any other great circle given in poſition in the ſame right line, whatever be the Sun's declination.

II. If a line, adjuſted parallel to the Earth's axis, be applied to the centre of a circle, the plane of which is given in poſition, the inclination of the line's ſhadow to the plane of the meridian is the meaſure of apparent time, allowing one hour for 15°, &c.

If the planes of declination circles interſect any plane given in poſition, and a line is applied to the latter plane at their common interſection, parallel to the Earth's axis, the ſhadow of the line thus applied will determine the apparent time of the day.

III. Let the axis of a ſphere, upon which the circles of declination are deſcribed, be directed to the pole of the heavens. The ſphere being biſected by the plane of any great circle, the ſection will be a dial, of which the axis of the ſphere is called the ſtyle.

The interſection of the dial with the meridian is marked XII. the interſection of the adjacent hour circle, inclined to the meridian, at an angle of 15° is marked I. and ſo on; the intermediate times being marked at the interſections of the correſponding circles of declination, with the plane of the dial. Since, when the hour angle is the ſame, the ſection of the ſtyle's ſhadow is the ſame, whatever be the Sun's declination; it follows, that whenever the Sun ſhines upon the ſtyle, the time of the day being given, the ſhadow will always interſect the dial in the ſame right line: this line therefore will determine the apparent time. To obtain the true time the equation muſt be applied with its proper ſign to the time denoted by a dial.

A dial is called vertical, horizontal, equatorial, &c. according as the plane of it coincides with that of a vertical circle, the horizon or equator in the heavens.

IV. The equatorial inſtrument conſiſts of three circles, the planes of which may be ſo adjuſted as to coincide with the planes of the horizon, equator, and any given circle of declination in the heavens reſpectively.

A teleſcope is applied to the declination circle, to the plane of which the line of collimation is parallel.

The moſt common uſes to which the equatorial inſtrument is applied, are as follow:

1. To determine the poſition of the meridian.
2. To find the apparent time of the day.
3. To direct the teleſcope to any point in the heavens, the right aſcenſion and declination of which are known; or to determine the right aſcenſion and declination of objects after having adjuſted the inſtrument to the latitude of the place.

V. Let the equatorial circle be inclined to the horizon, at an angle equal to the complement of the latitude to 90°. The inſtrument is adjuſted to the latitude.

The inclination of the equator in the heavens to the horizon, is equal to the complement of the latitude, to 90°.

VI. In order to determine the meridian, the declination circle muſt be ſet to the declination of a known ſtar. If the ſtar be brought into the centre of the field, by means of the equatorial and azimuth circles, the plane of the equatorial circle will coincide with that of the equator in the heavens (ſetting aſide the effects of refraction).

The plane of a ſecondary to the equatorial and azimuth circles will coincide with the plane of the meridian; to which the line of collimation will be directed, when the equatorial index points to XII.

VII. The inſtrument being adjuſted to the latitude, let the declination circle be ſet to the Sun's declination, correſponding to the time of obſervation: if the line of collimation be directed to the Sun's centre, by moving the equatorial and azimuth circles in their own planes, the arc of the equatorial circle intercepted between XII. and the point to which the index is directed, will determine the apparent time of the day.

In this caſe the equatorial circle becomes a dial, the plane of which is perpendicular to the ſtyle, or, as it is uſually called, an equatorial dial. It is to be obſerved, that in both the preceding articles, the apparent elevation of the celeſtial objects above their true places by refraction, will cauſe the poſition of the meridian, and the time deduced from obſervation with the equatorial teleſcope, to deviate ſomething from the truth. Theſe errors both in the hour angle and azimuth may be eſtimated thus:

Let $\dot{r}$ repreſent the apparent elevation of the ſtar, or Sun's centre, cauſed by refraction: S = the angle contained between a vertical circle and a circle of declination, paſſing through the ſtar : then the variation of the hour angle, cauſed by the apparent elevation of the ſtar $= \frac{\dot{r}}{\text{coſ. lat.} \times \text{ſin. azimuth}}$, and the contemporary variation in the azimuth $= \frac{\dot{r} \times \text{cotang. S}}{\text{ſin. ſtar's zenith diſt.}}$ Since by the properties of the equatorial inſtrument when adjuſted, the plane of the declination circle will always be vertical when the equatorial index points to XII. it follows that the poſition of the meridian found in Art. VI. will be corrected by moving the azimuth circle, in a direction contrary to the Sun's azimuth, through an arc $= \frac{\dot{r} \times \text{cot. S}}{\text{ſin. zen. diſt}}$.

It may be inferred from hence, that in the latitude 52° 12′ 40″, refraction will cauſe no error in the poſition of the meridian, determined as in Art. VI. when the obſerved ſtar's declination and altitude = 62° 45′.

In general, let a repreſent the latitude of a place, b the declination of an obſerved ſtar : find an angle, the ſine of which $= \frac{\text{ſin. } a}{\text{ſin. } b}$, and let this angle $= A$: if the obſervation for adjuſting the equatorial to the meridian is taken when the altitude of the obſerved ſtar is $= A$, the poſition of the meridian will be correct without allowing for refraction.

VIII. Let the inſtrument be adjuſted to the latitude and the meridian ; if the declination circle be ſet to the declination of any ſtar, &c. and the equatorial circle be moved in its own plane, the line of collimation will be always directed to the parallel of declination, which the ſtar deſcribes in its diurnal revolution.

If the right aſcenſion and declination of any ſtar be given, the line of collimation of the equatorial teleſcope may be directed to it; by which means Jupiter, Venus, and the fixed ſtars of the firſt magnitude may be obſerved in the day time.

IX. The altitude of an object above the horizon may be meaſured on the declination circle of the equatorial inſtrument.

When the equatorial index points to XII. the plane of the declination circle paſſes through the zenith; the line of collimation therefore being adjuſted horizontal, and afterwards directed to a ſtar, by moving the declination and azimuth circles in their own planes; the index of the declination circle will determine the arc of the vertical circle intercepted between the ſtar and the horizon.

X. The altitude of an object may be determined, by meaſuring the arc of a great circle, intercepted between the ſtar and the horizon, and the inclination of the ſame great circle to the horizon.

This is the method deſcribed in p. 198. Art. VI. and may be put in practice by means of the equatorial inſtrument thus: let the ſine of the eſtimated altitude of the object, be s; elevate the equatorial circle above the horizon to an angle, the ſine of which $= \sqrt{s}$, rad. being $= 1$. The declination circle being ſet to 0, direct the line of collimation to the ſtar, by the equatorial and azimuth circles, moved in their own planes; obſerve the arc of the equatorial circle, intercepted between the index and VI. if the ſine of this arc $= p$, the ſine of the obſerved altitude will be equal to $\sqrt{s} \times p$, radius being 1.

Thus, if the altitude of the Sun's lower limb were eſtimated to be about 37°. The proceſs for finding the altitude by this method will be as follows:

Log. ſine 37° = 9.7794630

$\times\frac{1}{2}$ = 9.8897315 = the ſine of 50° 53′ = the angle, of which the ſine is $= \sqrt{s}$: having therefore elevated the equatorial circle to this angle above the horizon, turn the equatorial and azimuth circles in their own planes, until the object is brought into the centre of the field: ſuppoſe the equatorial arc meaſured from VI. to be 51° 4′:

Log. ſin. 50°.53 = 9.8897850
Log. ſin. 51°.4 = 9.8909113

9.7806963 = log. ſin.

37° 7′ 22″, the true altitude.

XI. Suppoſe Q to repreſent the zenith, (vide fig. in Note to Art. VI. p. 198.) S the place of a ſtar, QS and QO the arcs of two vertical circles; SO, a great circle paſſing through the ſtar, and equal to QO. If the arc QO and the angle SOQ be determined by obſervation, the zenith diſtance SQ will be known.

This method of determining zenith diſtances is ſimilar to the indirect menſuration of altitudes before deſcribed, and is capable of great exactneſs, when the arc to be meaſured is ſmall, eſpecially ſince the line of collimation by the conſtruction of the equatorial inſtrument may be adjuſted to the zenith.

The equatorial inſtrument may be applied to this method by means of the following rules: let the ſine of half the eſtimated zenith diſtance $= s$; find an angle, the ſine of which $= \sqrt{s}$, radius being 1. Elevate the equatorial circle above the horizon to an angle, the ſine of which $= \sqrt{s}$; and ſet the declination circle to the complement of the ſame angle to 90°. By means of the equatorial and azimuth circles moved in their own planes direct the line of collimation to the ſtar: obſerve the arc intercepted between the equatorial index and XII. the ſine of half this arc $\times \sqrt{s}$, radius being $= 1$, will be the ſine of half the zenith diſtance required.

Or thus: let $\sqrt{s}$ be the ſine of the angle at which the equatorial circle is inclined to the horizon; and the manner of obſervation being the ſame as has juſt been deſcribed: let v be the verſed ſine of the equatorial arc meaſured from XII. then the verſed ſine of the zenith diſtance required will be $= vs$, radius being $= 1$.

Thus, ſuppoſe the zenith diſtance were eſtimated to be about 4°.

Log. ſin. 2° $= 8.5428192$

which being divided by 2 will $= 9.2714096 =$ ſine of about 10° 46′.

The equatorial circle being elevated above the horizon to an angle $= 10° \ 46'$, and the declination circle ſet to the complement of that angle, or 79° 14′ from the equator, let the obſervation be taken: if the equatorial arc eſtimated from XII. is $= 22° \ 10'$, the zenith diſtance required will be found thus:

$$
\begin{array}{rl}
\text{Log. ſine } 10° \ 46' = 9.2713997 = & \text{log. } \sqrt{s} \\
\times 2 & \\
\hline
8.5427994 = & \text{log. } s \\
\text{Log. verſ. ſin. } 22° \ 10' = 8.8687018 & \\
\hline
7.4115012 = & \text{log. verſed ſine of}
\end{array}
$$

4° 6′ 51″.4, the true zenith diſtance.

XII. To meaſure any ſmall vertical angles by the equatorial inſtrument.

An angle which does not exceed a few minutes cannot be meaſured directly by the graduated arc of a ſector or circle, becauſe errors which are inconſiderable in reſpect of larger angles become great in proportion to thoſe which are very ſmall.

The following method of meaſuring ſmall vertical angles with the equatorial inſtrument is very exact, and on ſeveral occaſions may be found uſeful to obſervers. Let the elevation of the equatorial circle above the horizon be a; the polar diſtance of the teleſcope (either greater or leſs than 90°) be $=b$. Let the index of the equatorial circle point to XII. (correſponding to noon), when the line of collimation is directed to any object. Suppoſe that the equatorial circle deſcribes an angle of which the verſed ſine is v; then if the azimuth circle is moved in its plane, ſo that the line of collimation may be returned to the vertical in which the firſt obſerved object was ſituated, it will have been depreſſed through a ſmall arc of the vertical which will be $= \frac{v \times \text{ſine } a \times \text{ſine } b}{\text{ſine } \overline{b-a}}$ to radius 1. If $b = 90°$, that is if the line of collimation is parallel to the plane of the equatorial circle, the depreſſion will be meaſured by an arc $= v \times$ tangent a to radius 1.

One uſe of this method is to eſtimate with great exactneſs the angle ſubtended by the horizontal wire of the teleſcope at the centre of the object glaſs: having ſelected any well-defined fixed object at a ſufficient diſtance, let the declination circle be ſet to 0: elevate the equatorial circle ſo as to bring the object into the field of the teleſcope when the index points to XII. (merid.) on the equatorial: let the upper edge of the horizontal wire be brought to coincide with the fixed object by altering the elevation of the equatorial circle: by moving the equatorial and azimuth circles in their own planes let the lower edge of the wire be brought to coincide with the fixed object. Suppoſe the elevation of the equatorial was 41° 3′, the angle denoted by the index of the equatorial $= 55'$ then the angle ſubtended by the wire at the centre of the object glaſs, will be meaſured by an arc = vers. ſine 57′ × tang. 41° 3′ = 24″.6.

By the application of this rule the vertical diameter of a planet may be meaſured: ſuppoſe the planet near its greateſt altitude on the eaſt ſide of the meridian: the declination circle being ſet to 0, and the equatorial circle to XII. (merid.) let the upper edge of the horizontal wire be brought to coincide with the upper limb of the planet (or lower limb as it appears in the teleſcope) by altering the elevation of the equatorial circle: by turning the equatorial and azimuth circles in their own planes let the lower edge of the wire be brought to coincide with the lower limb of the planet (that is with the upper limb as it appears in the field of the teleſcope): ſuppoſe v to be the verſed ſine of the angle the equatorial circle has been moved through, and let the inclination of the equatorial circle to the horizon be a: then the ſum of the diameters of the planet and wire will be an angle which is meaſured by an arc = v × tang. a, from which ſubtracting the diameter of the wire before found, the diameter of the planet will be obtained.

XIII. An exact method of meaſuring the altitudes of objects by the equatorial inſtrument.

Let the equatorial circle be elevated above the horizon to any angle = a: let the line of collimation be adjuſted horizontal, when the index of the equatorial points to XII. (correſponding to midnight). If the equatorial circle is turned in its own plane through an angle the verſed ſine of which $=v$, the line of collimation will be inclined to the horizon at an angle the ſine of which $= \frac{v}{2} \times$ ſine $2a$; this is the correct value of the altitude of an object to which the line of collimation is directed. Thus, ſuppoſe the elevation of the equatorial circle above the horizon were 30° 7′ and the arc through which the equatorial has moved from XII. = 40° 2′; then the ſine of the altitude obſerved will be = $\frac{\text{vers. ſine } 40^\circ\ 2'}{2}$ × ſine 60° 14′: the proceſs will be thus:

Log. vers. ſine 40° 2′ = 9.3698272
Log. $\frac{1}{2}$ = 9.6989700
Log. ſine 60° 14′ = 9.9385470

9.0073442 = ſine of 5° 50′ 14″ the altitude of the object obſerved.

The quantity *a* or the elevation of the equatorial circle above the horizon may be of any value, ſo that it is greater than ½ the angle to be meaſured; ſince when *a* is given, the greateſt angle which can be obſerved by this method is that of which the ſine is = ſine $2a$.

XIV. Let the equatorial circle be elevated above the horizon to an angle of 45°: when the equatorial index points to XII. (correſponding to midnight) let the teleſcope be adjuſted horizontal: if by turning the equatorial and azimuth circles through an angle of which the verſed ſine $= v$, any object be brought into the centre of the field, the ſine of its altitude above the horizon will be $= \frac{v}{2}$.

In this caſe *a* or the elevation of the equatorial circle above the horizon is 45° and conſequently $2a = 90°$ of which the ſine = 1 wherefore the ſine of the obſerved altitude will be = $\frac{v}{2}$; which may be calculated without trouble, and will afford a very exact meaſure of altitudes and depreſſions, much beyond what can be obtained by direct obſervation with an inſtrument of equal radius.

Suppoſe an object to be obſerved in this manner, the arc on the equatorial circle being 20° 2′.

The proceſs for inferring the altitude of the object will be this:

Log. verſed ſine of 20° 2′ = 8.7818022

Log. $\frac{1}{2}$ = 9.6989700

8.4807722 = ſine of 1° 44′ 1″ the altitude of the obſerved object.

To form ſome eſtimation how far this obſervation may be affected by errors in adjuſting the equatorial circle to 45°, or in reading off the equatorial arc, ſuppoſe both of theſe arcs to be too ſmall by 1′; and let the reſult be deduced from the rule contained in the laſt article. In this caſe the ſine of the altitude will be =

$$\frac{\text{verſed ſine of } 20^\circ\ 1'}{2} \times \text{ſine } 89^\circ\ 58'.$$

Log. verſed ſine 20° 1′ = 8.7810866

Log. $\frac{1}{2}$ = 9.6989700

Log. ſine 89° 58′ = 9.9999999

8.4800565 = ſine of 1° 43′ 51″ differing no more than 10″ from the former reſult.

In this computation it is obſervable that an error of 1′ in the inclination of the equatorial circle to the horizon, cauſes a difference of only a ſingle unit in the laſt decimal place of the logarithmic ſine of the obſerved altitude; and if the variation of the inclination was increaſed even to 10′, no ſenſible error would be occaſioned by it in the reſult:

if the angle at which the equatorial circle is inclined to the horizon be aſſumed leſs than 45°, the error from the variation of inclination may become ſenſible; and though the error occaſioned by a given variation of the equatorial arc is at the ſame time decreaſed, yet the ſum of both will be greater than when the inclination of the equatorial circle to the horizon is 45°, if they ſhould combine in their effects to increaſe or diminiſh the obſerved angle.

The inclination of 45° will in general be far preferable to any other for the menſuration of altitudes by the method here propoſed. A ſmall table N° X. is inſerted in the Appendix, containing the elevations and depreſſions of objects above and below the horizon correſponding to given arcs of the equatorial circle meaſured from XII when the inclination of the equatorial circle to the horizon is 45°. By this table the exactneſs of the ſpirit level which is ſuſpended from the teleſcope of an equatorial inſtrument may be truly aſcertained without the trouble of calculation. Suppoſe for example; a motion of 40′ of the equatorial arc from XII. cauſes a perceptible alteration in the level: by the table it appears that the depreſſion is about 7″ and conſequently that the level is ſenſible to 7″. This table may be alſo applied to meaſure ſmall elevations and depreſſions of objects above and below the horizon; to the determination of horizontal refractions, &c.

Elevations of objects above the horizon, and depreſſions below it, are determined from the ſame rule; it being remembered, that when an object above the horizon is obſerved, the equatorial arc muſt be eſtimated from XII. correſponding to midnight: in obſerving depreſſions below the horizon, the equatorial arc muſt be meaſured from XII. correſponding to noon.

XV. To obtain a very exact meaſure of the horizontal angles ſubtended by fixed objects.

Let the equatorial circle be adjuſted parallel to the azimuth or horizontal circle: having directed the line of collimation to one of the objects, let the other object be brought into the centre of the field by moving the equatorial circle in its plane and the declination circle if neceſſary: let the line of collimation be returned to the firſt object by moving the azimuth circle and the declination circle in its own plane if neceſſary: if theſe obſervations be repeated any number of times, the angle through which the equatorial circle has been moved being divided by the number of times either of the two objects has been obſerved, will give an exact meaſure of the horizontal angle ſubtended by them: if the angle deſcribed by the azimuth circle in the contrary direction be divided by the ſame number, the reſult will be another value of the angle ſubtended by the objects: a mean of theſe two reſults will be very near the truth. Vid. a paper on the barometrical menſuration of altitudes by Sir George Shuckburgh Bart. Philoſophical Tranſactions for 1777.

OF PARALLAX, AND THE DETERMINATION OF INACCESSIBLE DISTANCES.

I. Let a line be drawn perpendicular to the diſtance between an adjacent object and any given ſtation: the apparent places of the object, when viewed from the extremities of the line, will be different.

The perpendicular line above deſcribed, is called a baſe.

2. The extremities of the baſe are called ſtations.

3. The angle ſubtended by the extremities of the baſe at the object is called the angle of parallax.

4. The baſe is to the leſſer of the two diſtances of the object from the extremities of the baſe, as the tangent of parallax to radius; and to the greater, as the ſine of the ſame angle to radius.

II. Suppoſe lines to be drawn from the two ſtations to an object: one of the angles contained by theſe lines, and the baſe being a right angle, the other will be the complement of the parallax to 90°.

If the angles at the ſtations terminating a given baſe be known, the parallax, and conſequently the diſtance of the object, may be determined.

III. If the diſtance of an object be greater than 100000 times the baſe, the angles at the ſtations will not ſenſibly differ from two right ones; the lines drawn from the object to the ſtations are therefore, phyſically ſpeaking, parallel.

One of the angles at the baſe, in the propoſitions contained in this ſection, is ſuppoſed to be 90°.

The moſt accurate inſtruments conſtructed for the menſuration of angles, cannot be depended upon, to leſs than 2″, the tangent of which is to radius as 1 to 103132.

The angle the tangent of which is to radius as 1 to 100000, is 2″.06, or little more than two ſeconds.

IV. The parallax of an object, the diſtance of which is above 100000 times greater than that between the two ſtations of obſervation, is inſenſible.

If the object be at a greater diſtance from either ſtation, than 100000 times the baſe, the angle at one of the ſtations being 90°, the angle at the other will be more than 89° 59′ 57″.9; the difference of which angle and 90° being ſcarcely more than 2″, is two ſmall to become ſenſible by obſervation.

V. If the parallax of an object (obſerved with an inſtrument ſufficiently exact to meaſure an angle of 2″) be inſenſible, the diſtance of it from either ſtation cannot be leſs than 100000 times the baſe, from the extremities of which it is obſerved.

It is to be remarked, that, although the diſtance of the object cannot be leſs than 100000 times the baſe, yet it may be greater, in any aſſignable ratio.

Lines drawn from any given points in a baſe, to an object, may be eſteemed in practice parallel, without ſenſible error, if the diſtance of the object is more than 100000 times the baſe.

Rays diverging from any point of the Sun's diſk, upon the ſurface of the Earth, may be eſteemed parallel, if their diſtance from each other exceed not about 1000 miles, at the Earth's ſurface; becauſe 1000 miles is to the Earth's diſtance from the Sun, in a proportion of little more than that of one to 100000.

In the ſame manner, plumb lines hanging freely and at reſt, not being diſtant more than 68 yards from each other, may be accounted parallel.

Rays diverging from a fixed ſtar, to any parts of the Earth's orbit, are, phyſically ſpeaking, parallel, becauſe the parallax of the Earth's orbit, ſeen at the ſtar, is leſs than 2″.

Two planes drawn parallel to each other, and paſſing through the extremities of a diameter of the Earth's orbit, if produced, will appear to coincide with the ſame great circle of the heavens; becauſe the diameter of the Earth's orbit when ſeen from the fixed ſtar, ſubtends an angle leſs than 2″. In like manner, if a plane, paſſing through the Earth's centre, be parallel to a plane drawn to the ſurface, theſe planes, when produced, apparently coincide with the ſame great circle in the heavens.

OF THE SOLAR SYSTEM.

I. The Solar Syſtem conſiſts of the Sun, and the Planets and Comets revolving about it.

II. There are ſeven primary planets; viz. Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgium Sidus.

This new planet was diſcovered by Mr. Herſchel in the year 1781.

The planets revolve according to the order of the ſigns, in orbits nearly circular, round the Sun as a centre.

The apparent annual revolution of the Sun in the ecliptic ariſes from the real motion of the Earth in its orbit; Venus and Mercury revolve in orbits which are contained within that of the Earth, and are called Inferior Planets; Mars, Jupiter, Saturn, and the Georgium Sidus, which revolve in orbits exterior to the Earth's orbit, are called Superior Planets.

The orbits of all the planets are elliptical, but the principal phenomena, which demonſtrate the truth of the Copernican Syſtem, are the ſame, whether the orbits are conſidered as elliptical or circular. The latter ſuppoſition is uſually adopted in giving a general deſcription of the diſpoſition and motion of the heavenly bodies.

III. The planes of the orbits in which the planets revolve are inclined differently to the plane of the ecliptic, but none ſo much as 8°.

IV. Three of the primary planets have ſatellites or ſecondary planets revolving round them as centres, in orbits nearly circular, and accompanying them in their revolutions round the Sun.

V. The Moon is the only ſatellite attendant upon the Earth, and performs its periodic revolution in about 27 days 7 hours, deſcribing an orbit which is inclined to the plane of the ecliptic at an angle of about 5°.

If the periodic times of any two of the heavenly bodies be a and b, the ſynodical period will $= \frac{ab}{a-b}$, ſuppoſing their orbits to be circular, and their motion uniform.

The periodical time of the Moon $= 27.29$; the periodic time of the Sun's apparent revolution $= 365.25$ days; wherefore a ſynodical month $=$

$$\frac{365.25 \times 27.29}{365.25 - 27.29} = 29.49$$ or about 29 days and a half.

VI. The Moon is obſerved to turn the ſame face towards the Earth in every part of her revolution.

The Moon turns round her axis, which is nearly perpendicular to the plane of her orbit, in the ſame time with that in which ſhe compleats her revolution round the Earth.

VII. Four ſatellites revolve round Jupiter, and five round Saturn.

VIII. Saturn is encompaſſed with a luminous ring, the appearance of which uaries according to the different poſitions of the ſpectator.

IX. The planets and their ſecondaries are opake and ſpherical bodies.

X. The Earth turns round its axis which is inclined to the plane of the ecliptic at an angle of 66° 32′, compleating its revolution in $23^h\ 56^m\ 4^s$.

The apparent diurnal revolution of the Sun and ſtars from eaſt to weſt, is cauſed by the revolution of the Earth round its axis from weſt to eaſt.

If the Sun's place in the ecliptic were always the ſame, 360° of the equator would paſs under a given meridian between the time of the Sun's leaving and returning to the ſame meridian. But ſince the mean increaſe of tre Sun's right aſcenſion in 24 hours is 59′ 8″, it follows, that, during 24 hours, or a mean ſolar day, 360° 59′ 8″ of the equator will paſs under a given meridian: whence we derive the time of the Earth's revolution round its axis by the following proportion: as 360° 59′ 8″ to 360° ſo is 24^h to 23^h 56^m 4^s, the time ſought.

The great circle of the heavens, in the plane of which the Earth performs its diurnal revolution, is called the equator.

The different ſeaſons of the year are cauſed by the inclination of the ecliptic to the equator.

XI. Other planets have been obſerved to revolve round their axes.

The diurnal revolutions of Jupiter, Mars, and Venus have been diſcovered from the ſpots which are viſible on their diſks; any one of theſe ſpots being obſerved, appears to revolve, returning to the ſame poſition which it has departed from at equal intervals of time.

It is diſcovered from the ſpots on the diſk, that the Sun turns round his axis in about 25 days.

XII. The Earth is ſurrounded with a thin tranſparent fluid, called collectively the Atmoſphere.

The Atmoſphere is the cauſe of the twilight, the apparent elevation of the ſtars above their true places, and the general diffuſion of the Sun's rays.

XIII. Comets are a ſort of planets which revolve in very excentric ellipſes round the Sun, which is ſituated in one of the foci.

Comets are obſerved to revolve, both according to the order of the ſigns, and in the contrary direction.

The planes of their orbits are inclined to the plane of the ecliptic at various angles, not being confined within any limit.

PHENOMENA WHICH DEMONSTRATE THE TRUTH OF THE COPERNICAN SYSTEM, AND SOME COROLLARIES.

I. Venus and Mercury are never ſeen in oppoſition to the Sun.

A planet is ſaid to be in oppoſition, when the longitudes of the Sun and planet differ by 180°.

II. The angle of elongation of an inferior planet from the Sun never exceeds a certain limit.

III. The ſuperior planets are obſerved to be ſometimes in oppoſition, and ſometimes in conjunction.

IV. The ſuperior planets are retrograde in oppoſition, and progreſſive in conjunction.

V. There is a certain angle of elongation from the Sun, at which a planet appears ſtationary when ſeen from the Earth.

Let the periodic time of a ſuperior planet be to that of an inferior as $1 : t$, and let their diſtances from the Sun be as $1 : s$. An angle, the coſine of which $= \sqrt{\frac{1-s^2}{1-t^2}}$ radius being 1, will be the angle of elongation from the Sun, at which the inferior planet appears ſtationary when obſerved from the ſuperior. The angle of elongation of a ſuperior planet, when ſeen ſtationary from an inferior, is obtuſe; and the ſine of it $= \sqrt{\frac{s^2 - t^2}{1 - t^2}} \times \frac{1}{s}$ — radius being 1.

It is here ſuppoſed, that the planets revolve uniformly in the ſame planes, and in circles, the common centre of which coincides with the centre of the Sun.

The obſervation of the planet's ſtations will therefore deviate ſomething from the preceding rules.

VI. A ſpectator on the Earth ſees that part of a planet's diſk only which is contained between the circles of viſion and illumation.

The Moon and the inferior planets appear horned when they are ſituated between the Earth and Sun.

The circles of viſion and illumination always ſo nearly coincide in Jupiter and Saturn, that theſe planets appear in all ſituations to ſhine with a full diſk.

In Mars, when near quadrature, theſe circles being conſiderably inclined to each other, cauſe a part of the illuminated diſk to be intercepted from our view; in this poſition Mars is ſaid to be gibbous.

VII. The parallax of the Sun is about 8″½.

This is the parallax of the Sun when viewed in the zenith and in the horizon, or the horizontal parallax; wherefore the ſemidiameter of the Earth is to the diſtance of the Earth from the Sun, as the ſine of 8″½ to radius, or as 1 : 24266 ∴ and ſince the radius of the Earth = 3970 miles, the diſtance of the Sun from the Earth = 24266 × 3970 = near 97000000 miles.

VIII. The parallax of the fixed ſtars is not ſenſible.

The parallax is therefore leſs 2″, and conſequently the diſtance of the ſtar greater than 100000 times the baſe from the extremities of which it is obſerved, that is, greater than 100000 times the diameter of the Earth's orbit, or greater than 100000 × 194000000 miles.

IX. The parallax of a fixed ſtar being not more than 2″, the Sun when viewed from a ſtar would appear under an angle leſs than $\frac{32' \; 6''}{200000}$, or leſs than the hundredth part of a ſecond, and therefore could not be diſtinguiſhed from a point.

Since bodies equal in magnitude and ſplendor to the Sun, being placed at the diſtance of the fixed ſtars, would appear to us as the ſtars now do, it may be ſuppoſed probable, that the fixed ſtars are bodies ſimilar to the Sun, which is the centre of the Solar Syſtem.

This being the caſe, the reaſon will appear why a fixed ſtar, when viewed through a teleſcope magnifying two hundred times, appears no other than a point. For the apparent diameter of the ſtar being leſs than one hundredth part of a ſecond, when magnified two hundred times, will ſubtend an angle of not ſo much as 2″ at the eye of a ſpectator obſerving it in the teleſcope.

The parallax of the fixed ſtar, when viewed from the oppoſite parts of the Earth's orbit, is here aſſumed 2″; but it is probable that the parallax of the neareſt ſtar is much leſs, and conſequently the diſtance greater in the ſame proportion, as the parallax is leſs.

X. The appearance of the fixed ſtars in the heavens will be in every reſpect the ſame, in whatever part of the Earth's orbit, the ſpectator is ſituated.

XI. The change of place or parallax of the Moon, when ſeen in the zenith and the horizon, is, at a medium, 57′.

XII. Comets have no ſenſible diurnal parallax.

The diſtance of comets muſt be greater than that of the Moon from the Earth.

XIII. Comets are obſerved to change their places in the heavens, on account of the Earth's motion in its orbit.

Comets, when viſible, are either within the regions of the Solar Syſtem, or not very far diſtant from the orbit of Saturn.

CONCERNING ECLIPSES.

I. The planets and their ſecondaries ſhine by the light of the Sun reflected from them.

II. An opake body intervening between a planet and the Sun, will intercept part, or even all the rays of light from arriving at the planet, according to the magnitude and proximity of the opake body.

Any opake ſphere ſubtending an angle at the ſpectator's eye, equal to, or greater than, the apparent diameter of the Sun, will totally intercept the rays of light, if the centre of it be in a line joining the centre of the Sun and the eye of the ſpectator.

III. The ſhadow of the Earth is of a determined length, and will totally eclipſe the Sun from any part of the Moon's ſurface ſituated in it.

The figure of the Earth's ſhadow is a cone; the axis of which is a continuation of the line, which joins the centres of the Earth and Sun; the Earth being between the Sun and the vertex of the ſhadow.

The length of a planet's ſhadow is to the ſemi-diameter of the planet, as radius to the ſine of the Sun's apparent ſemi-diameter.

IV. Suppoſe the poſition of the ſhadow to be reverſed, ſo that the vertex of the cone may be between the Sun and the planet, the axis remaining in the ſame direction as before; a continuation of this cone increaſing, *ſine limite*, is called the penumbra.

VI. If a ſpectator be ſituated in the penumbra of a planet, a part of the Sun's light will be intercepted from his view.

VI. The Sun is never eclipſed but at the time of a New Moon, and the Moon is never eclipſed but when at the Full.

VII. Eclipſes of the Moon and Sun would happen at every Full and New Moon, if the plane of the Moon's orbit were coincident with the plane of the ecliptic.

VIII. The plane of the Moon's orbit is inclined to that of the ecliptic, at an angle of about 5°.

Thd line in which theſe planes interſect each other, is called the line of the nodes.

IX. If the angular diſtance of the Moon's centre from the node, at the time of the New Moon, be leſs than about ſixteen degrees and a half, there will be an eclipſe of the Sun.

X. If the angular diſtance of the Full Moon's centre from the node, be leſs than about twelve degrees and a half, the Moon will be eclipſed.

The limits of ſolar, being greater than thoſe of lunar eclipſes, there will happen in a given time a greater number of eclipſes of the Sun than of the Moon. But an eclipſe of the Moon is viſible to the inhabitants of one half of the globe; whereas a ſolar eclipſe can be obſerved on a ſmall proportion of the Earth's ſurface only: For this reaſon, at a given place, more lunar eclipſes will be viſible in a certain time, than eclipſes of the Sun.

XI. No primary planet can fall into the ſhadow of another primary planet; but part of a planet's ſurface may be eclipſed by falling into the ſhadow of its ſecondary.

Any planet may fall into the penumbra of an inferior planet.

XII. When the Earth falls into the penumbra of Venus or Mercury, the rays of the Sun are in part intercepted from our view; and the planet, in paſſing over the Sun, appears as a dark ſpot upon the diſk.

XIII. Mercury and Venus would be obſerved to paſs over the diſk of the Sun at every inferior conjunction, if the planes of their orbits coincided with that of the ecliptic.

OF THE PHENOMENA WHICH ARISE FROM THE EXCENTRICITY OF THE EARTH'S ORBIT, AND THE DIVISION OF TIME.

I. THE apparent diameter and hourly motion of the Sun are obſerved to vary as the Earth revolves in its orbit: and the variations are ſuch as demonſtrate the orbit to be elliptical.

If the Earth's orbit be ſuppoſed an excentric circle, according to the hypotheſis of ſome antient aſtronomers, deductions formed from this hypotheſis diſagree with obſervation.

If the Earth be ſuppoſed to revolve in an ellipſe round the Sun placed in one of the Foci, whatever deductions are formed from this ſuppoſition, preciſely agree with Phenomena.

II. Suppoſing the excentricity and ſemiaxis major of the Earth's orbit to be known, if the Sun's place in the ecliptic be given, the apparent ſemi-diameter of the Sun being determined.

In any ellipſe let t = the ſemi-axis major,
c = the ſemi-axis major,
m = the excentricity.

Let a point be aſſumed in the circumference of the ellipſe, and let a line be drawn joining the point aſſumed and the focus. Let l = the coſine of the angle contained between the line drawn as above and the axis major: then the diſtance of the aſſumed point from the focus $= \frac{c^2}{t \pm lm}$. It is inferred from hence, that if d'' be the apparent ſemi-diameter of the Sun when at its mean diſtance from the Earth, in any other poſition in which the coſine of the Sun's longitude from the apogee $= l$, the apparent ſemi-diameter will $= d''t \times \frac{t \pm lm}{c^2}$.

The obſervation of the Sun's apparent ſemi-diameter agrees accurately with the preceding deduction, formed à priori from the elliptic hypotheſis, of which this agreement is a confirmation.

d'' Being the hourly apparent motion of the Sun in the ecliptic when at the mean diſtance, the hourly motion when the Sun is in any point of the ecliptic, the coſine of the longitude from the apogee being l, will $= d'' t^2 \times \frac{\overline{t \pm lm}^2}{c^4}$.

Theſe concluſions, and any other derived in a ſimilar manner for the apparent ſemi-diameter, and hourly motion of the Sun at a given longitude, do not differ ſenſibly from obſervation, which is (among others) a demonſtration that the orbit in which the Earth revolves is an ellipſe, the Sun being in one of the foci.

III. A line joining the Earth and Sun deſcribes equal areas in equal times during the revolution of the Earth in its orbit.

Since the interſection of the ecliptic and the equator divides the area of the Earth's orbit unequally, it follows that the times of the Sun's entering the vernal and autumnal equinoxes, will divide the year unequally.

The daily increaſe of the Sun's right aſcenſion is variable on two accounts. 1. The variation of the Sun's angular velocity in the ecliptic. 2. The inclination of the ecliptic to the equator.

IV. The tropical year is the time elapſed between the Sun's leaving either of the tropics, and returning to the ſame tropic.

The tropical year conſiſts of 365 days, 5 hours, and about 49 minutes.

IV. The ſidereal year is the time elapſed between the Sun's leaving any fixed ſtar in the ecliptic, and returning to the ſame ſtar.

The ſidereal year conſiſts of 365 days, 6 hours, and about 9 minutes.

From this difference between the tropical and ſidereal years, it is inferred that the equinoxial points change their places in the heavens, revolving in a direction contrary to the order of the ſigns at the rate of about 1° in 72 years. The colures will be ſubject to the ſame motion, and the pole of the equator will deſcribe a leſſer circle round the pole of the ecliptic at 23° 28′ from it, and will compleat its revolution in $72 \times 360 = 25920$ years.

The variation of the equator's inclination to the ecliptic is not here conſidered.

VI. The ſolar year is not compoſed of any exact number of days.

Hence ariſes the neceſſity of applying a frequent correction to the calendar, leſt the ſeaſons of the year ſhould ceaſe to correſpond with the Sun's place in the ecliptic, reckoning from the equinoxes or tropics.

The tropical year conſiſts of 365 days, 5 hours, and 49 minutes; if, therefore, it were accounted only 365 days, according to the cuſtom of the Egyptians, the deficient 5 hours and 49 minutes would in 4 years accumulate to a difference of 23 hours, 16 minutes, by which the calendar would precede the Sun; but if, in order to compenſate for this, a whole day be inſerted into the calendar every 4th year this will be allowing too much by 44 minutes, which renders it neceſſary to leave out a day about every hundredth year.

CONCERNING THE PROGRESSIVE MOTION OF LIGHT.

I. WHEN Jupiter's ſatellites emerge from the ſhadow of their primary, the light reflected from them after their obſcuration is obſerved to arrive at the Earth about ſixteen minutes ſooner when Jupiter is in oppoſition, than when he is near conjunction.

A ray of light paſſes over the Earth's orbit in about ſixteen minutes, or nine hundred and ſixty ſeconds of time, and conſequently moves at

the

the rate of $\frac{194000000}{960}$, or about 202083 miles in a ſecond.

II. The fixed ſtars appear to deſcribe ellipſes round points which are ſituated in one of the foci of the ellipſes.

This apparent motion in the fixed ſtars is cauſed by the progreſſive motion of light, and the motion of the Earth in its orbit.

The longer axes of the ellipſes are parallel to the plane of the ecliptic, and ſubtend angles which are all equal to about 40″ and the ſhorter axes = 40″ × ſine ſtar's latitude.

The apparent latitude of a given ſtar is leaſt when the Sun is 90° before the ſtar in longitude.

The apparent longitude is greateſt when the ſtar and Sun differ in longitude by 180°, and is the leaſt when the longitudes of the Sun and the ſtar are the ſame.

III. The velocity of light is to the velocity of the Earth in its orbit, as radius is to the tangent of 20″, or as 10313 to 1.

This proportion is deduced from Dr. Bradley's theory of the ſtar's aberrations; from the eclipſes of Jupiter's ſatellites, it is inferred, that the velocity of light is to that of the Earth in its orbit, as 202083 to 19.31, or as 10465 to 1: the near agreement of theſe proportions, is a confirmation of the principle aſſumed for the explication of the phenomena.

CONCERNING THE CAUSES OF THE PLANETS MOTION.

I. THE primary planets gravitate towards the Sun, and the ſecondary planets towards their reſpective primaries.

A projectile force having been impreſſed upon a planet when at reſt, would cauſe it to proceed uniformly in a right line for ever, unleſs the Sun, by the continual action of its attractive power, deflected the planet from its rectilinear direction. This attraction is called a centripetal force.

The centripetal force varies with the diſtance of the planets from the Sun, ſo that a planet at half the diſtance would be attracted by a quadruple force; at one third of the diſtance, the force of attraction would be nine times greater.

II. The planets revolve in ellipſes very little differing from circles, round the common centre of gravity of the planets and the Sun.

The Sun ſo much exceeds the planets in quantity of matter, that the centre of gravity of the Sun, and all the planets, however ſituated, is very near the Sun itſelf.

The areas deſcribed by the line joining a planet, and the point to which the centripetal force is always directed, are proportional to the time of a planet's motion in its orbit.

Some of the properties of central forces may be illuſtrated experimentally.

III. Suppoſe two bodies, urged by centripetal and projectile forces, to deſcribe circles round a centre of force ſituated in the common centre of the circles; if the periodic times be the ſame, the centripetal or centrifugal forces will be in the ſame ratio as the radii of the circles.

IV. If the periodic times be directly as the radii, the forces will be inverſely as the radii.

V. If the ſquares of the periodic time be as the cubes of the diſtances, the centripetal forces will be inverſely as the ſquares of the diſtances.

This caſe obtains in the revolutions of the planets and their ſecondaries, except that the planets and their ſecondaries perform their motions in ellipſes; whereas in the experiment, the bodies revolve in circular orbits; but the ſame properties are demonſtrated (as far as regards this article) when the orbits are ellipſes as when they are circular, the radius of the circle being changed for the greater ſemi-axis of the ellipſe.

VI. The ſpheriodal figure of the Earth is cauſed by the revolution of the Earth round its axis.

The Earth is an oblate ſpheroid, an equatorial diameter being greater than the polar, in the proportion of 230 : 229.

This figure of the Earth, the inclination of the equator to the ecliptic, and the attraction of the Sun and Moon, cauſe the preceſſion of the equinoxes.

VII. The centrifugal force of the Earth revolving round the common centre of gravity of the Earth and Moon, contributes to increaſe the tides in thoſe parts of the Earth which are moſt remote from the Moon.

VIII. The lunar irregularities are cauſed by the action of the Sun upon the Earth and Moon.

APPENDIX.

TABLE I.

For converting Degrees, Minutes, and Seconds of the Equator into Apparent Time.

° '	h. m.	m. s.	° '	h. m.	m. s.	ſec.		ſec.	
1	0	4	30	2	00	1	0.1	30	2.0
2	0	8	40	2	40	2	0.1	40	2.7
3	0	12	50	3	20	3	0.2	50	3.3
4	0	16	60	4	00	4	0.3		
5	0	20	70	5	40	5	0.3		
6	0	24	80	5	20	6	0.4		
7	0	28	90	6	00	7	0.4		
8	0	32	100	6	40	8	0.5		
9	0	36	200	13	20	9	0.6		
10	0	40	300	18	40	10	0.6		
20	1	20				20	1.3		

TABLE II.

For converting Hours, Minutes, and Seconds of Apparent Time into Degrees, Minutes, and Seconds of the Equator.

hours,	°	minutes, ſeconds.	° '	' "	ſeconds,	"
1	15	1		15	·1	1.5
2	30	2		30	.2	3.
3	45	3		45	·3	4.5
4	60	4	1	00	·4	6.
5	75	5	1	15	·5	7.5
6	90	6	1	30	.6	9.
7	105	7	1	45	·7	10.5
8	120	8	2	00	.8	12.
9	135	9	2	15	·9	13.5
10	150	10	2	30		
11	165	20	5	00		
12	180	30	7	30		
14	210	40	10	00		
20	300	50	12	30		

TABLE III.

For aſſigning the Angles ſubtended by Arcs of given lengths to radius = 1.

Arcs expreſſed in parts of radius.	Angles ſubtended. ″
.1	20626.48
.2	41252.96
.3	61879.44
.4	82505.92
.5	103132.40
.6	123758.88
.7	144385.36
.8	165011.84
.9	185638.32

TABLE IV.

For aſſigning the Arcs expreſſed in parts of radius = 1, ſubtended by given Angles.

Angles ſubtended. ″	Length of Arcs expreſſed in parts of radius.
1	.00000.48481
2	.00000 96963
3	.00001.45444
4	.00001.93925
5	.00002.42407
6	.00002.90888
7	.00003.39370
8	.00003 87851
9	.00004.34332

TABLE V.

The Refractions of the Heavenly Bodies in Altitude. Vide Tables requiſite to be uſed with the Nautical Ephemeris, p. 1.

App. Alt.	Refrac.	App. Alt.	Refrac.	App. Alt.	Refrac.	App. Alt.	Refrac.	App. Alt.	Refrac.
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 0	33 0	1 40	20 18	3 20	13 34	6 30	7 51	9 50	5 20
0 5	32 10	1 45	19 51	3 25	13 20	6 40	7 40	10 0	5 15
0 10	31 22	1 50	19 25	3 30	13 6	6 50	7 30	10 15	5 7
0 15	30 35	1 55	19 0	3 40	12 40	7 0	7 20	10 30	5 0
0 20	29 50	2 0	18 35	3 50	12 15	7 10	7 11	10 45	4 53
0 25	29 6	2 5	18 11	4 0	11 51	7 20	7 2	11 0	4 47
0 30	28 22	2 10	17 48	4 10	11 29	7 30	6 53	11 15	4 40
0 35	27 41	2 15	17 26	4 20	11 8	7 40	6 45	11 30	4 34
0 40	27 0	2 20	17 4	4 30	10 48	7 50	6 37	11 45	4 29
0 45	26 20	2 25	16 44	4 40	10 29	8 0	6 29	12 0	4 23
0 50	25 42	2 30	16 24	4 50	10 11	8 10	6 22	12 20	4 16
0 55	25 5	2 35	16 4	5 0	9 54	8 20	6 15	12 40	4 9
1 0	24 29	2 40	15 45	5 10	9 38	8 30	6 8	13 0	4 3
1 5	23 54	2 45	15 27	5 20	9 22	8 40	6 1	13 20	3 57
1 10	23 20	2 50	15 9	5 30	9 8	8 50	5 55	13 40	3 51
1 15	22 47	2 55	14 52	5 40	8 54	9 0	5 48	14 0	3 45
1 20	22 15	3 0	14 36	5 50	8 41	9 10	5 42	14 20	3 40
1 25	21 44	3 5	14 20	6 0	8 28	9 20	5 36	14 40	3 35
1 30	21 15	3 10	14 4	6 10	8 15	9 30	5 31	15 0	3 30
1 35	20 46	3 15	13 49	6 20	8 3	9 40	5 25	15 30	3 24

App. Alt.		Refrac.		App. Alt.	Refrac.		App. Alt.	Refrac.		App. Alt.	Refrac.	
°	′	′	″	°	′	″	°	′	″	°	′	″
16	0	3	17	30	1	38	50	0	48	70	0	21
16	30	3	10	31	1	35	51	0	46	71	0	19
17	0	3	4	32	1	31	52	0	44	72	0	18
17	30	2	59	33	1	28	53	0	43	73	0	17
18	0	2	54	34	1	24	54	0	41	74	0	16
18	30	2	49	35	1	21	55	0	40	75	0	15
19	0	2	44	36	1	18	56	0	38	76	0	14
19	30	2	39	37	1	16	57	0	37	77	0	13
20	0	2	35	38	1	13	58	0	35	78	0	12
20	30	2	31	39	1	10	59	0	34	79	0	11
21	0	2	27	40	1	8	60	0	33	80	0	10
21	30	2	24	41	1	5	61	0	32	81	0	9
22	0	2	20	42	1	3	62	0	30	82	0	8
23	0	2	14	43	1	1	63	0	29	83	0	7
24	0	2	7	44	0	59	64	0	28	84	0	6
25	0	2	2	45	0	57	65	0	26	85	0	5
26	0	1	56	46	0	55	66	0	25	86	0	4
27	0	1	51	47	0	53	67	0	24	87	0	3
28	0	1	47	48	0	51	68	0	23	88	0	2
29	0	1	42	49	0	49	69	0	22	89	0	1

TABLE VI.

For the Dip or Depreſſion of the viſible Horizon at Sea below the true Horizon.

Height of the Obſerver's Eye in Feet.	Depreſſion or Dip of the Horizon.	
	′	″
1	1	4
2	1	30
3	1	50
4	2	7
5	2	22
6	2	36
7	2	48
8	3	0
9	3	11
10	3	21
20	4	45
30	5	49
40	6	43
50	7	31
60	8	14
70	8	53
80	9	30
90	10	4
100	10	37

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TABLE VII.

For expreſſing Hours, Minutes, and Seconds in Parts of a Day.

H.	Pts. of a Day.	H.	Pts. of a Day.	Min.	Pts. of a Day.	Sec.	Pts. of a Day.
1	.0417	13	.5417	1	.00069	10	.00012
2	.0833	14	.5833	2	.00139	20	.00023
3	.1250	15	.6250	3	.00208	30	.00035
4	.1667	16	.6667	4	.00278	40	.00046
5	.2083	17	.7083	5	.00347	50	.00058
6	.2500	18	.7500	6	.00417		
7	.2917	19	.7917	7	.00486		
8	.3333	20	.8333	8	.00555		
9	.3750	21	.8750	9	.00625		
10	.4167	22	.9167				
11	.4583	23	.9583				
12	.5000						

TABLE VIII.

Containing proportional Parts and Multiples of $3^m 55^s.91$.

H.	m.	ſec.	m.	ſec.	ſec.	ſec.	Days.	H.	m.	s.
1	0	9.8	1	0.2	10	0.0	1	0	3	55.9
2	0	19.7	2	0.3	20	0.1	2	0	7	51.8
3	0	29.5	3	0.5	30	0.1	3	0	11	47.7
4	0	39.3	4	0.7	40	0.1	4	0	15	43.6
5	0	49.1	5	0.8	50	0.1	5	0	19	39.6
6	0	59.0	6	1.0			6	0	23	35.5
7	1	8.8	7	1.2			7	0	27	31.4
8	1	18.6	8	1.3			8	0	31	27.3
9	1	28.5	9	1.5			9	0	35	23.2
10	1	38.3	10	1.6			10	0	39	19.1
20	3	16.6	20	3.3			20	1	18	38.2
			30	4.9			30	1	57	57.3
			40	6.6			40	2	37	16.4
			50	8.2			50	3	16	35.5
							60	3	55	54.6
							70	4	35	13.7
							80	5	14	32.8
							90	5	53	51.9
							100	6	33	10.9
							200	13	6	21.9
							300	19	39	32.8

TABLE IX.

Containing proportional Parts and Multiples of 3m56^s.555^s

H.	m.	ſec.	m.	ſec.	ſec.	ſec.	Days.	H.	m.	s.
1	0	9.9	1	0.2	10	.0	1	0	3	56.6
2	0	19.7	2	0.3	20	.1	2	0	7	53.1
3	0	29.6	3	0.5	30	.1	3	0	11	49.7
4	0	39.4	4	0 7	40	.1	4	0	15	46.2
5	0	49.3	5	0.8	50	.1	5	0	19	42.8
6	0	59.1	6	1.0			6	0	23	39.3
7	1	9.0	7	1.2			7	0	27	35.8
8	1	18.8	8	1.3			8	0	31	32.4
9	1	28.7	9	1.5			9	0	35	29.0
10	1	38.6	10	1.6			10	0	39	25.5
20	3	17.1	20	3.3			20	1	18	51.1
			30	4.9			30	1	58	16.7
			40	6.6			40	2	37	42.2
			50	8.2			50	3	27	7.8
							60	3	56	33.3
							70	4	45	58.8
							80	5	25	24.4
							90	5	54	50.0
							100	6	34	15.5
							200	13	8	31.1
							300	19	42	46.6

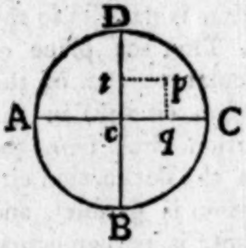
TABLE X.

For eſtimating ſmall Altitudes of the Teleſcope above the Horizon or Depreſſions below it, by Obſervations taken with an Equatorial Inſtrument, when the Equatorial Circle is inclined to the Horizon at an Angle of 45°.

Angular motion of the Equatorial Circle in its own plane from XII.		Altitudes or depreſſions.				Altitudes or depreſſion.				Altitudes or depreſſion.	
Arcs. °	'	'	"	Arcs. °	'	'	"	Arcs. °	'	'	"
0	10	0	0.4	1	40	0	43.6	4	10	4	32.6
0	20	0	1.7	1	50	0	52.8	4	20	4	54.8
0	30	0	3.9	2	0	1	2.8	4	30	5	17.9
0	40	0	7.0	2	10	1	13.7	4	40	5	41.9
0	50	0	10.9	2	20	1	25.5	4	50	6	6.7
1	0	0	15.7	2	30	1	38.2	5	0	6	32.5
1	10	0	21.4	2	40	1	51.7	5	10	6	59.0
1	20	0	27.9	2	50	2	6.1	5	20	7	26.5
1	30	0	35.3	3	0	2	21.3	5	30	7	54.8
				3	10	2	37.5	5	40	8	24.0
				3	20	2	54.5	5	50	8	54.0
				3	30	3	12.4	6	0	9	25.0
				3	40	3	31.1				
				3	50	3	50.7				
				4	0	4	11.2				

A METHOD OF ADJUSTING THE EQUATORIAL INSTRUMENT.

Fig. I.



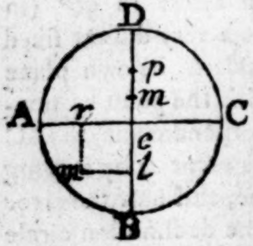
I. LET the circle ABCD repreſent the field of a teleſcope: *c*, the point in the field through which the line of collimation paſſes. Let a line AC paſſing through *c*, perpendicular to the axis of the teleſcope be adjuſted parallel to the plane of the declination circle, and ſuppoſe BD to be drawn through *c* perpendicular to AC and to the axis of the teleſcope.

II. Having fixed the declination circle to 90° (in either quadrant) direct the point *c* to a very diſtant fixed object *p*, and turn the equatorial circle in its own plane 180°. If *p* does not now coincide with the point *c*, imagine the lines *pq*, *pt* to be drawn perpendicular to AC and DB reſpectively: correct half the error *cq* by moving the declination circle in its own plane, and half the error *pq*, by altering the inclination of the declination circle to the plane of the equatorial. If the point *c* be now directed to the diſtant object *p*, and the equatorial circle be turned 180° in its own plane, the points *p* and *c* will coincide. By a very diſtant object is meant any viſible fixed point, the diſtance of which is not leſs than 400000 times the perpendicular diſtance of the line of collimation from the axis of the declination circle.

III. Set the declination nonius to 90°; and having turned the declination circle 180° in its own plane, direct the point *c* to the diſtant object *p*; and move the equatorial circle in its own plane 180°. If *p* now deviates from *c*, the error *cq* will ſhew double the error in the diviſion of the declination circle, and cannot be corrected by adjuſtment: ſuppoſe therefore the point *p* to be ſomewhere in the line DB, and correct ¼ of the error *tc* by altering the inclination of the declination circle to the plane of the equatorial, and ¼ of the ſame error *tc* by moving the line of collimation perpendicular to the plane of the declination circle.

When the preceding adjuſtments are correct, it appears, 1. That the line of collimation is parallel to the plane of the declination circle. 2. That the plane of the declination circle is perpendicular to that of the equatorial. 3. The errors of diviſion (if any) in the two oppoſite points of the declination circle (*viz.* the two 90°) are diſcovered. 4. When the declination circle is ſet to 0, the line of collimation is parallel, and when ſet to 90°, (in either quadrant) is perpendicular to the plane of the equatorial circle.

Fig. II.



IV. The azimuth circle being adjuſted horizontal, make AC parallel to it by turning the equatorial and azimuth circles in their own planes, and having directed the point *c* to be a very diſtant fixed object *p*, eſtimated to be in the horizon or near it, move the azimuth and declination circles in their own planes 180° each, ſo that the point *p* may appear ſomewhere in the line DB: if *p* deviates from *c*, biſect *pc* in *m*, the line of collimation being directed to *m*, will be horizontal. Let *m* be marked on the horizon.

V. Set the declination circle to 0, and turn the azimuth circle in its own plane 90°, ſo as to bring the horizontal object *m* into the line DB, and direct the point *c* to *m*, by moving the equatorial circle in its own plane if neceſſary. The plane of the declination circle will now be horizontal, and that of the equatorial circle vertical.

VI. Turn the equatorial and azimuth circles in their own planes 180°. If *m* now deviates from *c*, *mr* will be the error in diviſion of the equatorial, and *ml* the error of diviſion in the azimuth circle. (*Fig.* II.)

VII. In order to elevate the equatorial circle abov the horizon to any propoſed inclination, ſet the decli nation circle (in the ſouthern quadrant) to the propoſed angle of elevation, and having brought XII. on the equatorial circle to point o on the index, direct the line of collimation to the horizontal point *m*, by altering the inclination of the equatorial circle to the horizon, and by turning the azimuth circle in its own plane.

VIII. In order to adjuſt the equatorial index, ſet the declination circle to o, and by moving the equatorial and azimuth circles in their own planes, direct the line of collimation to the horizontal point *m*, and ſet the equatorial index to VI.

IX. By means of the laſt adjuſtment, when the equatorial index points to XII. the plane of the declination circle will coincide with a ſecondary common to the equatorial and azimuth circles; upon this ſecondary therefore the inclination of thoſe circles to each other will be truly meaſured. The 7th adjuſtment may be repeated if it be thought neceſſary.

X. If the teleſcope magnifies 60 times, it follows from the preceding adjuſtments, that any inclination of the line of collimation, or of the equatorial axis to the plane of the declination circle muſt be diſcovered, if it exceeds 1″; and (if the levels by means of which the poſition of the azimuth circle is rectified be tolerably good) the point *m* (deſcribed in Art. 4) will be truly horizontal within 6″ or 8″.

When the plane of the equatorial circle coincides that of the equator on the heavens, the line of collimation will during the motion of the declination circle turning in its own plane, paſs through the pole of the equator, without ſenſible error, and will conſequently deſcribe a circle of declination in the heavens: when the equatorial index points to XII. the line above mentioned will deſcribe a ſecondary common to the equator and horizon, or the meridian of the place, and will therefore paſs through the pole of the equator and zenith.

The errors in the poſition of the meridian, and in the time deduced from obſervation with the equatorial inſtrument (deſcribed and eſtimated in page 226.) may be corrected thus. The teleſcope being of that ſort which repreſents the image inverted with reſpect to the object, it follows that the ſtar's apparent elevation from refraction will cauſe the image in the teleſcope to be depreſſed through a ſpace which ſubtends an angle at the centre of the object glaſs, equal to the ſtar's refraction in altitude. Suppoſe DB (*Fig.* II.) to be perpendicular to the line of collimation, and to be adjuſted to the plane of a vertical circle which paſſes through an obſerved ſtar: let *cl* repreſent the depreſſion of the ſtar's image above deſcribed, which may be determined, if the principal focal length of the object glaſs, and the ſtar's refraction in altitude be given: if *l* be now directed to the ſtar's apparent place, it is manifeſt, that the line of collimation of the inſtrument which joins the point *c* and the centre of the object glaſs, will be directed to the true place of the ſtar, whereby the effects of refraction upon the hour angle and poſition of the meridian are entirely removed.

This is one among many other improvements which have been applied to the conſtruction of the equatorial inſtrument by Mr. Ramſden.

F I N I S.

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ERRATA.

P.	L.	
2	31	for .053, read .048.
3	3 and 5	ibid ibid.
3	5	for 1.06, read .96.
11	23	for bodies, read body.
14	19	after 1 of .5 inſert $\left\{\begin{array}{l}4\text{ of }.1\\ 1\text{ of }.05\end{array}\right.$
19	15	dele in
30	4	dele then
42	5	$b-y$, read $b+y$.
46	10	for intermediately, read intermediate.
58	12	after deſcribing, inſert the.
84	12	dele but.
91	10	for 2.3058, read 2.30258.
102	24 and 26	after any, inſert given.
104	16	dele be and inſert *be* after *will* line 15.
175	26	for $n-1$, read $n+1$.
180	15	for intermediate, read determinate.
195	8	for 20265″, read 206265″.
200	12	for $r\times$ ſine 1°, read $r+$ ſine 1°.
229	29	for 51″.4, read 57″.8.
230	36	for 55′, read 57′.
249	2	for being, read may be.
429	25	for $\overline{t\pm lm}$, read $\overline{t\pm lm}^2$.

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